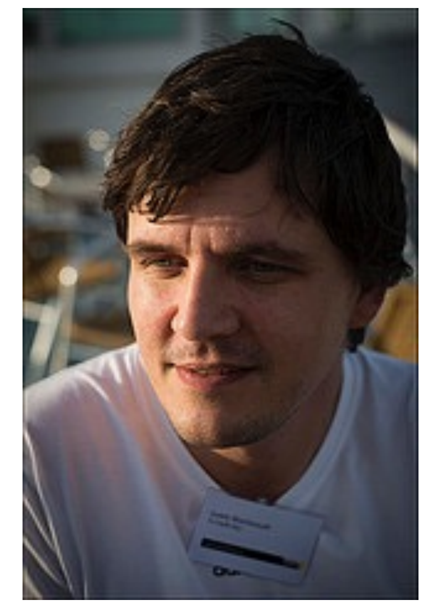
A Theory of Name Resolution

Pierre Neron[1]

Andrew Tolmach[2]

Eelco Visser[1]



Guido Wachsmuth[1]

[1] TU Delft — Delft University of Technology

[2] Portland State University

[ESOP15]

Static Name Resolution

```
module A {
    def s = 5
}

module B {
    import A
    def x = 6
    def y = 3 + s
    def f =
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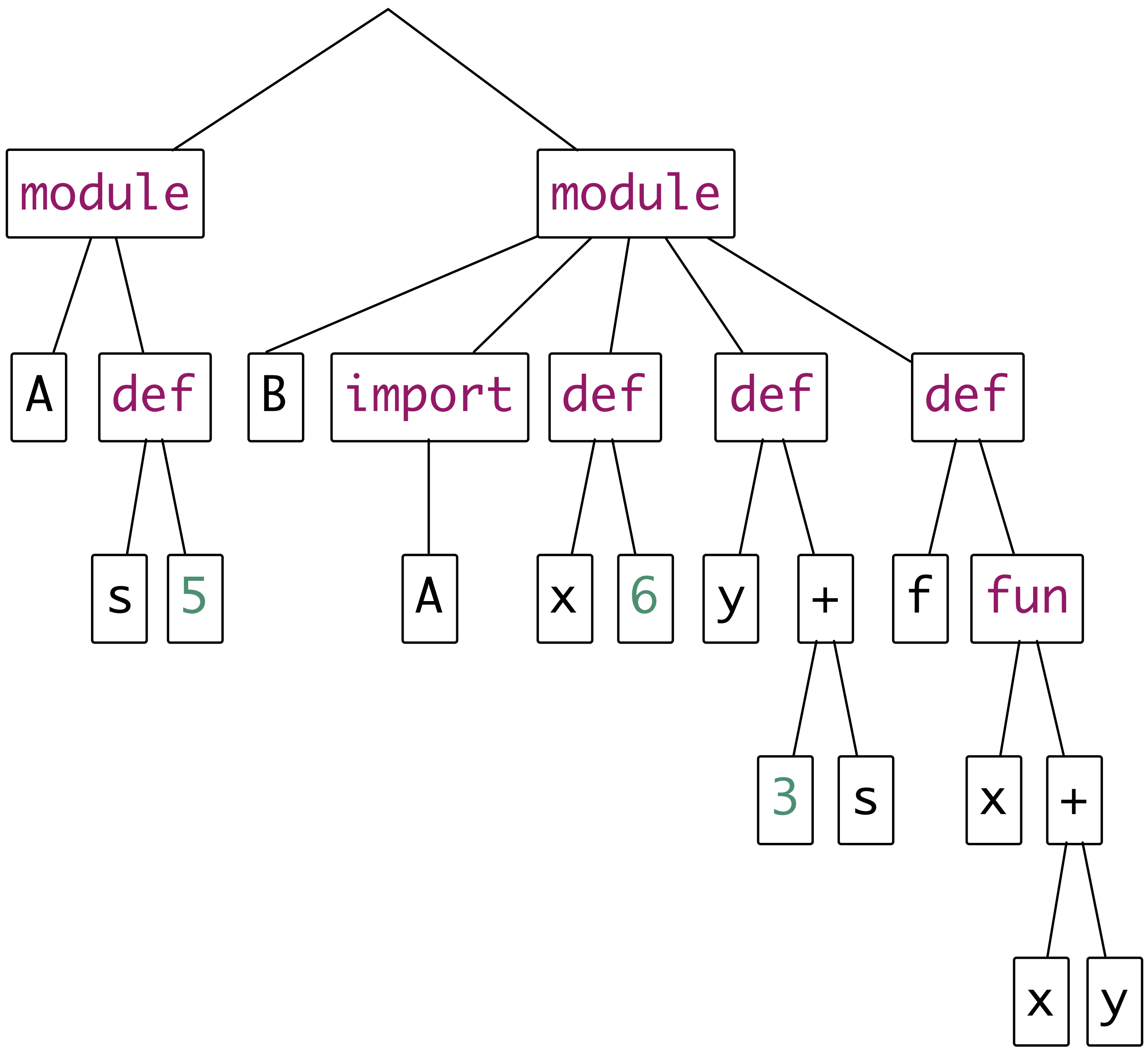

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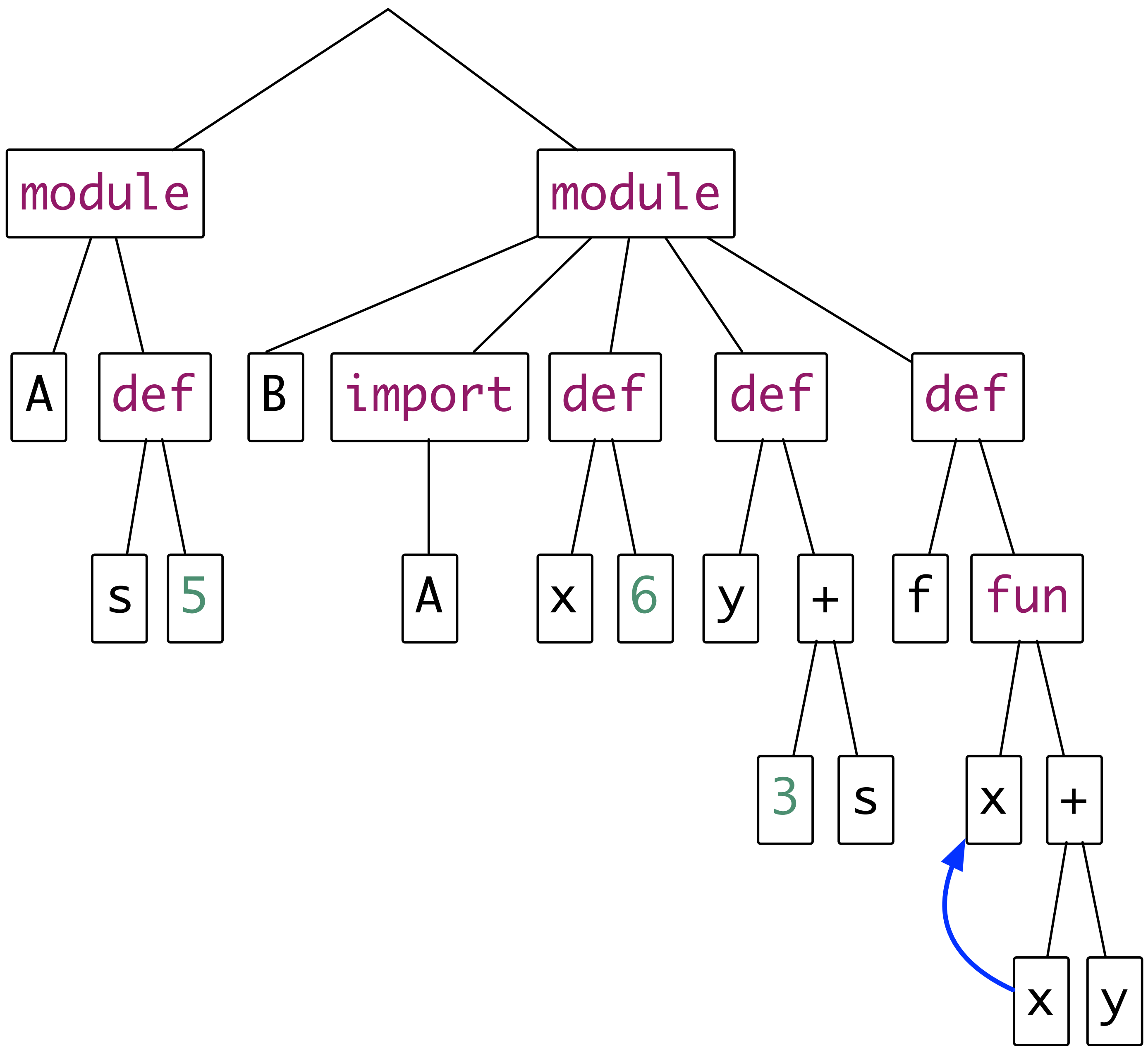

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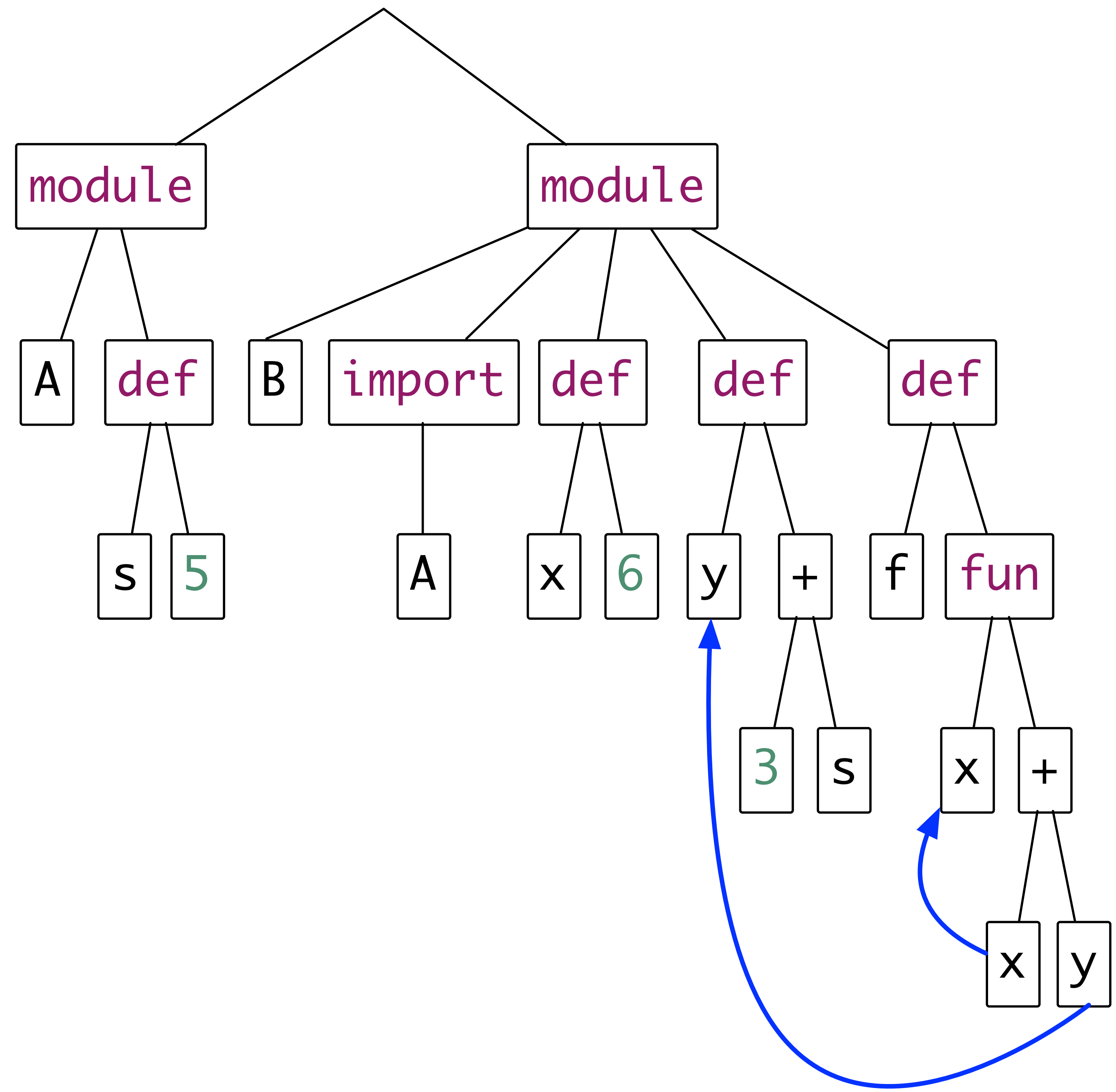

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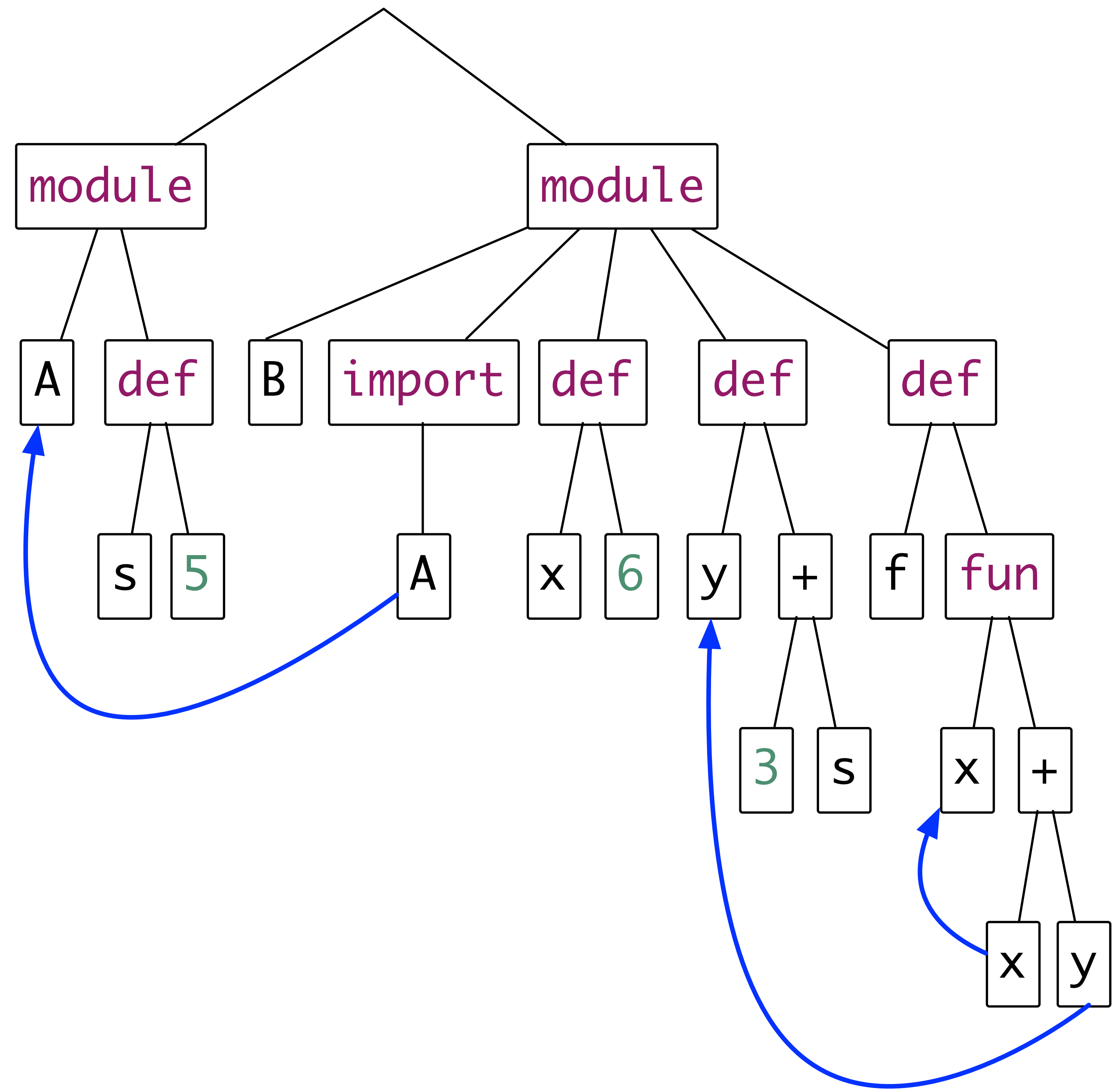

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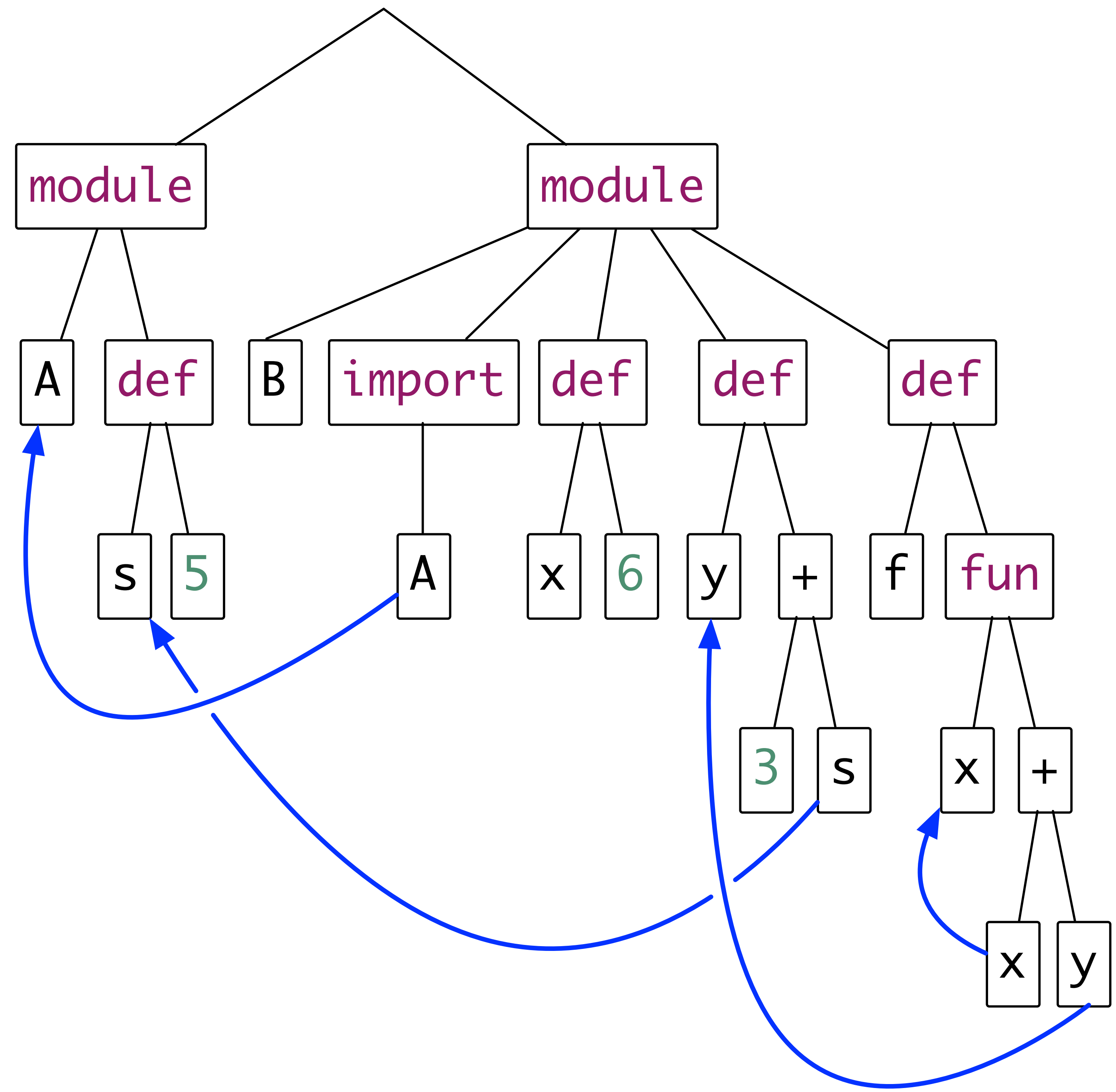

Use Scope Graphs to Describe Name Resolution

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Name Resolution is Pervasive

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Appears in many different artifacts…

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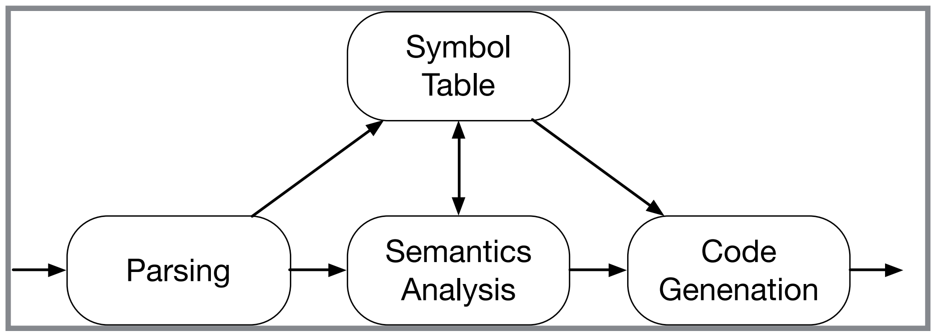

Compiler

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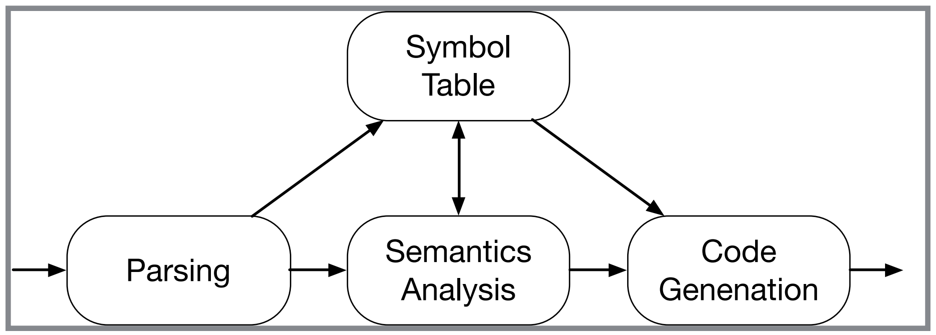

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$$\frac{x : \tau_1, \Gamma \vdash e : \tau_2}{\Gamma \vdash \lambda x.e : \tau_1 \to \tau_2}$$

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Semantics

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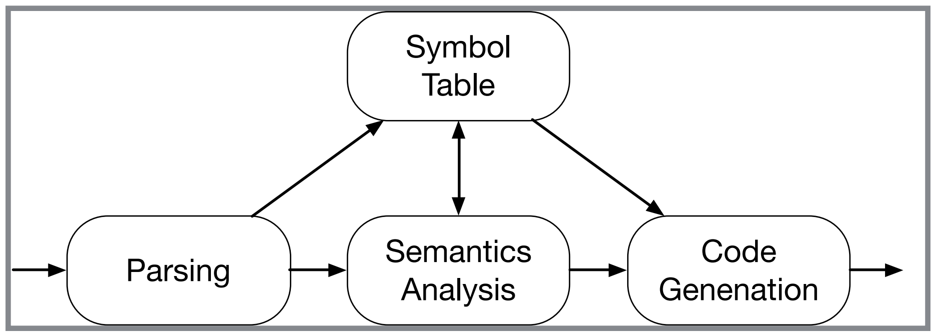

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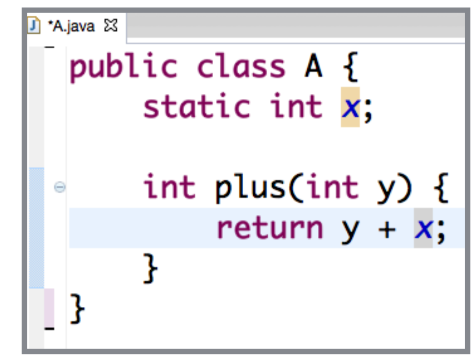

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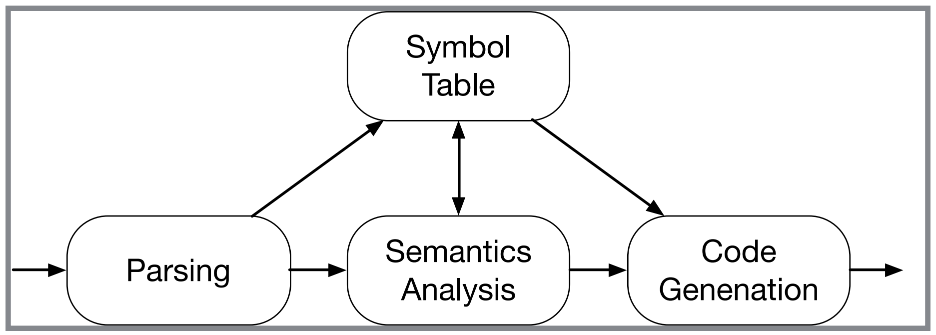

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*A.java

```
public class A {
    static int x;

    int plus(int y) {
        return y + x;
    }
}
```

Compiler

Semantics

IDE

… with rules encoded in many different ad-hoc ways

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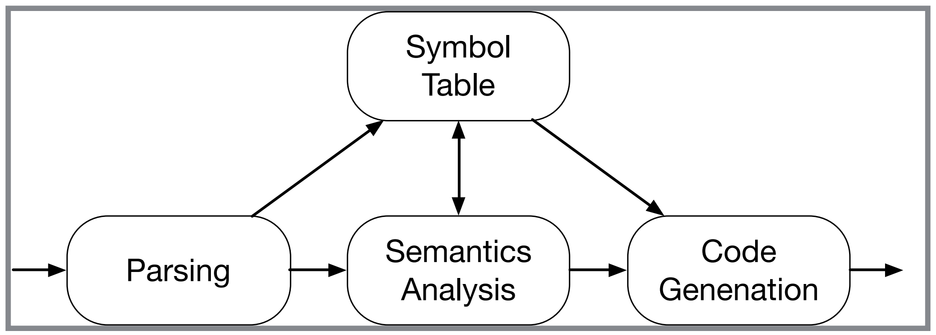

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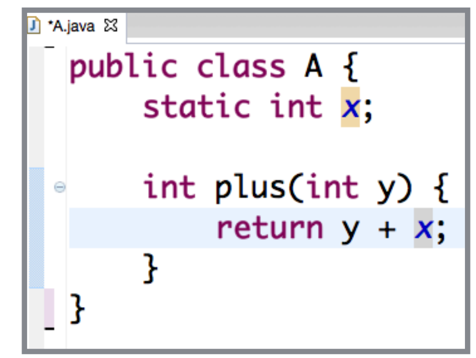

Compiler Semantics IDE

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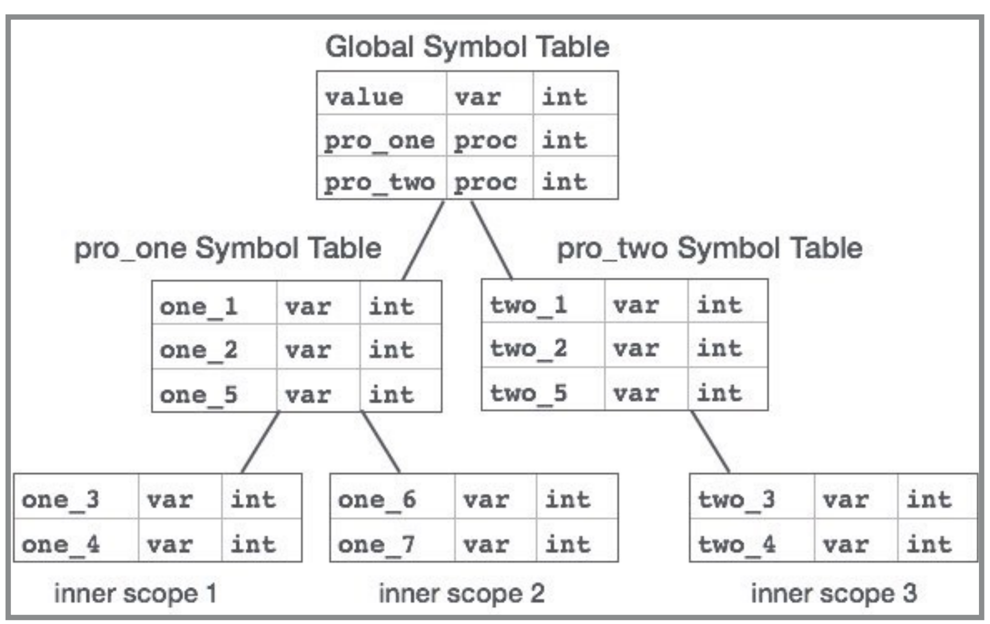

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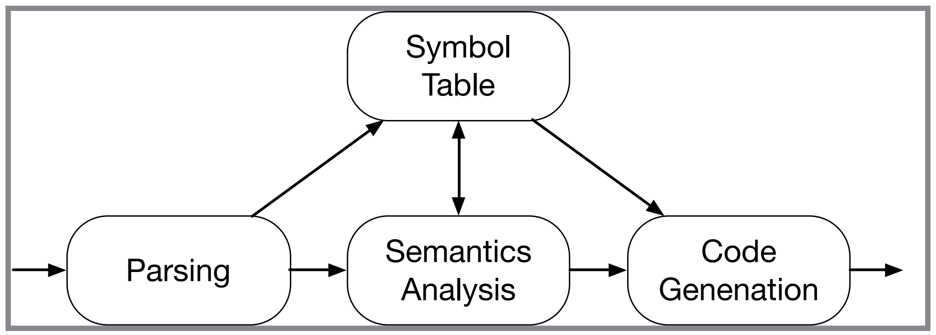

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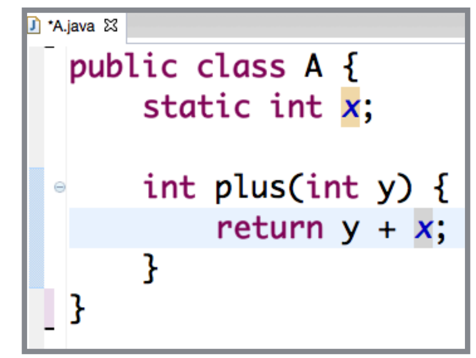

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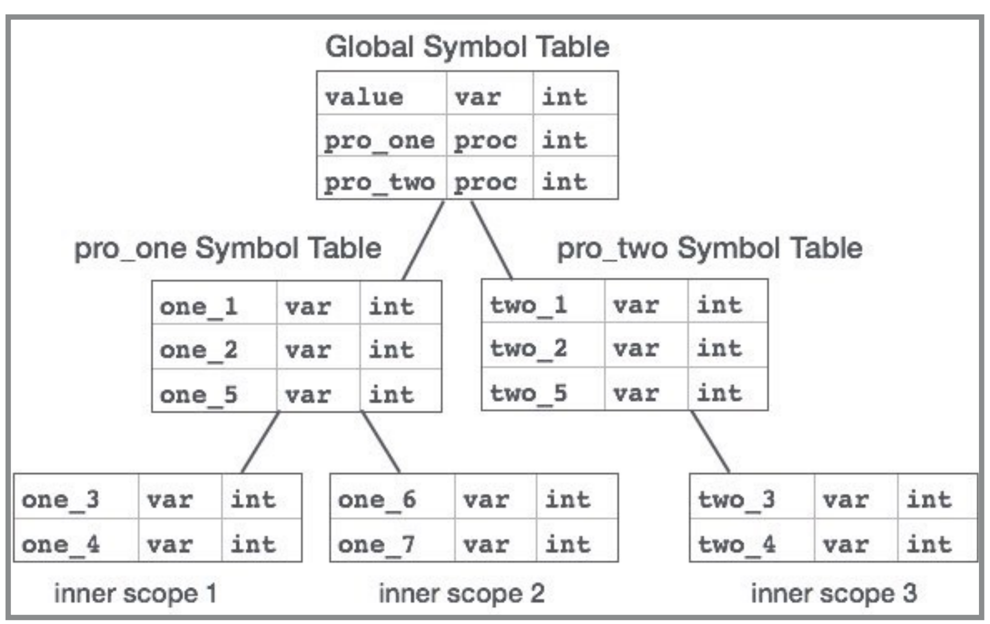

$\boxed{\texttt{x:int, }\Gamma}$

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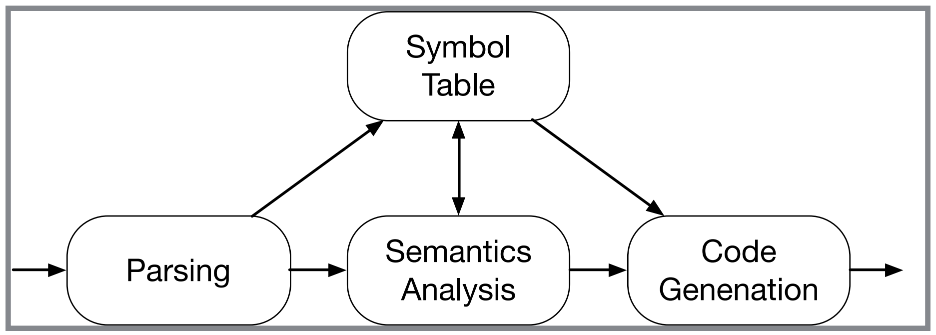

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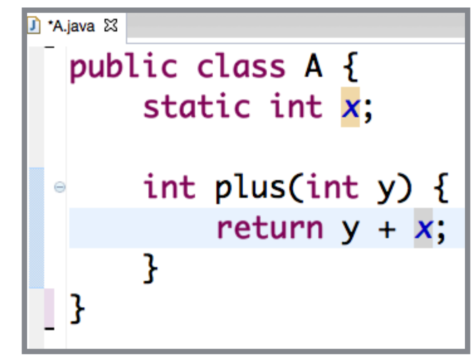

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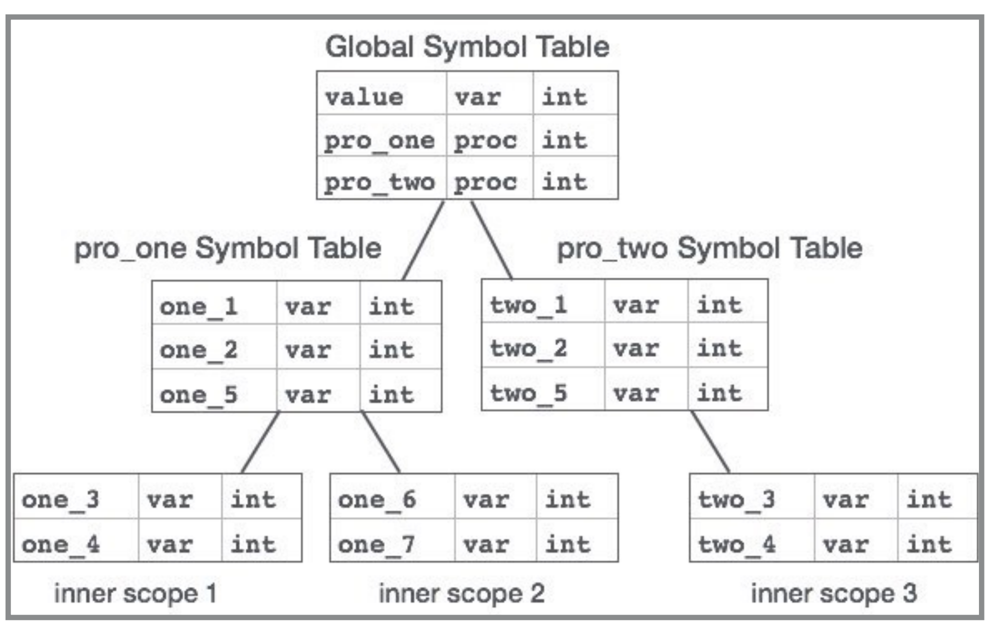

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`lookup(xᵢ)`

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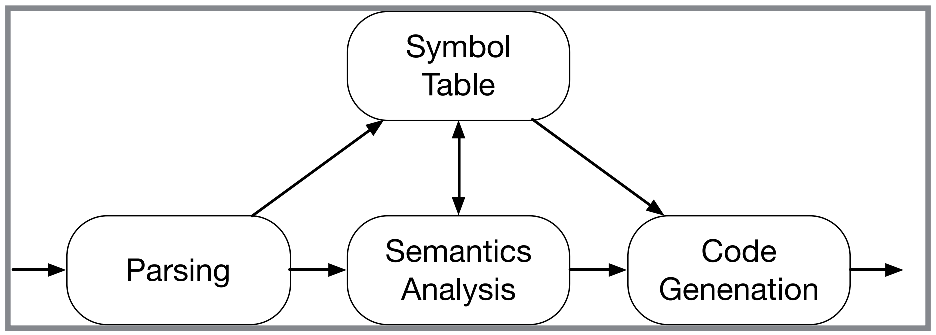

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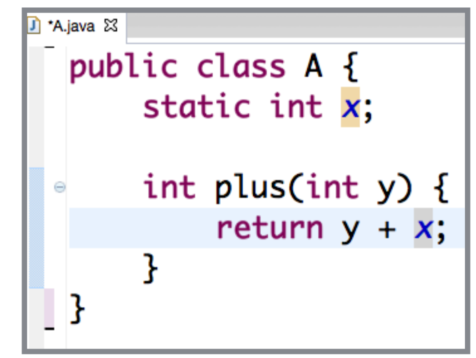

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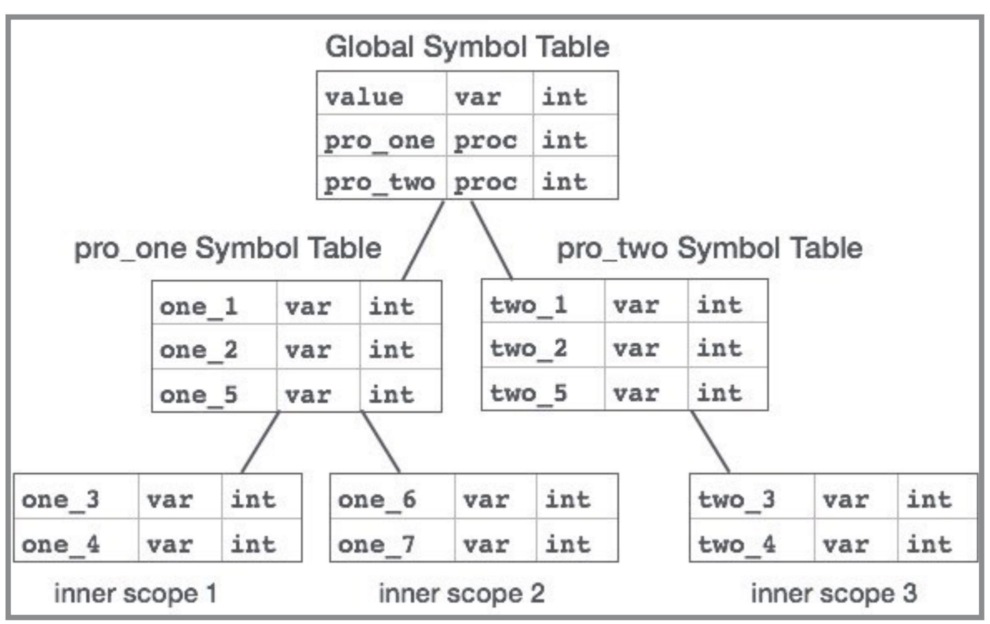

`x:int, Γ`

`lookup(x_i)`

`[3/x].σ`

No standard approach, no re-use

Contrast with Syntax

Contrast with Syntax

A standard formalism

Context-Free Grammars

Contrast with Syntax

A unique definition

A standard formalism

```
program  =  decl*
   decl  =  module id { decl* }
         |  import qid
         |  def id = exp
    exp  =  qid
         |  fun id { exp }
         |  fix id { exp }
         |  let bind* in exp
         |  letrec bind* in exp
         |  letpar bind* in exp
         |  exp exp
         |  exp ⊕ exp
         |  int
    qid  =  id
         |  id . qid
   bind  =  id = exp
```

Context-Free Grammars

Contrast with Syntax

A unique definition	*A standard formalism*	*Supports*

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Context-Free Grammars

Parser

AST

Pretty-Printing

Highlighting

Representing Bound Programs

Representing Bound Programs

- Many approaches to representing the results of name resolution within an (extended) AST, e.g.
  - numeric indexing [deBruijn72]
  - higher-order abstract syntax [PfenningElliott88]
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  - numeric indexing [deBruijn72]
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- Good support for binding-sensitive AST manipulation
- But: Do not say how to resolve identifiers in the first place!
  - Also: Can't represent ill-bound programs
  - And: Tend to be biased towards lambda-like bindings

Binding Specification Languages

Binding Specification Languages

- Many proposals for domain-specific languages (DSLs) for specifying binding structure of a (target) language, e.g.
  - Ott [Sewell+10]
  - Romeo [StansiferWand14]
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- Generate code to do resolution and record results
- But: what are the **semantics** of such a language?

The Missing Piece

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- Answer: the meaning of a binding specification for language L should be given by a function from L programs to their **“resolution structures”**

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- Answer: the meaning of a binding specification for language L should be given by a function from L programs to their **“resolution structures”**
- So we need a (uniform, language-independent) method for describing such resolution structures...
- ...that can be used to compute the resolution of each program identifier
  - (or to verify that a claimed resolution is valid)

Design Goals

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- Handle broad range of language binding features...
- ...using minimal number of constructs
- Make resolution structure language-independent
- Handle named collections of names (e.g. modules, classes, etc.) within the theory
- Allow description of programs with resolution errors

A Theory of Name Resolution

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*For **statically lexically scoped** languages*

A Theory of Name Resolution

For ***statically lexically scoped*** *languages*

A standard formalism

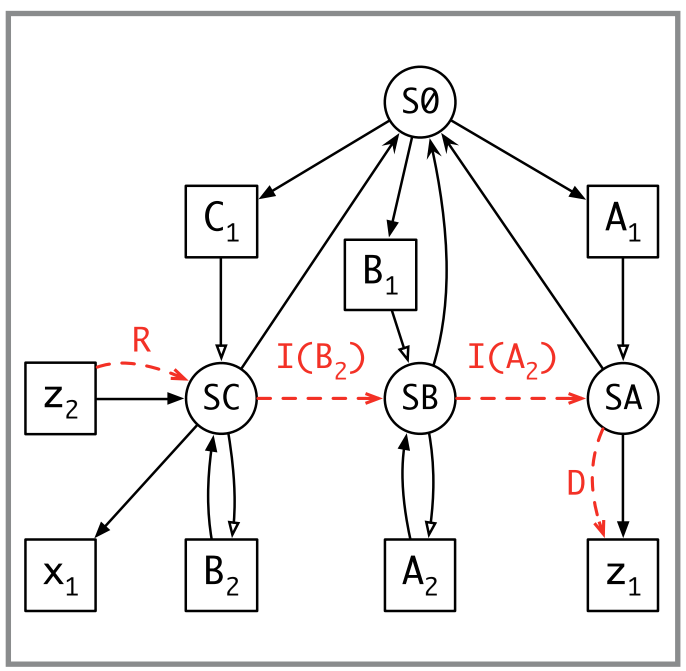
Scope Graphs

A Theory of Name Resolution

For ***statically lexically scoped*** *languages*

A unique representation

A standard formalism



Scope Graphs

A Theory of Name Resolution

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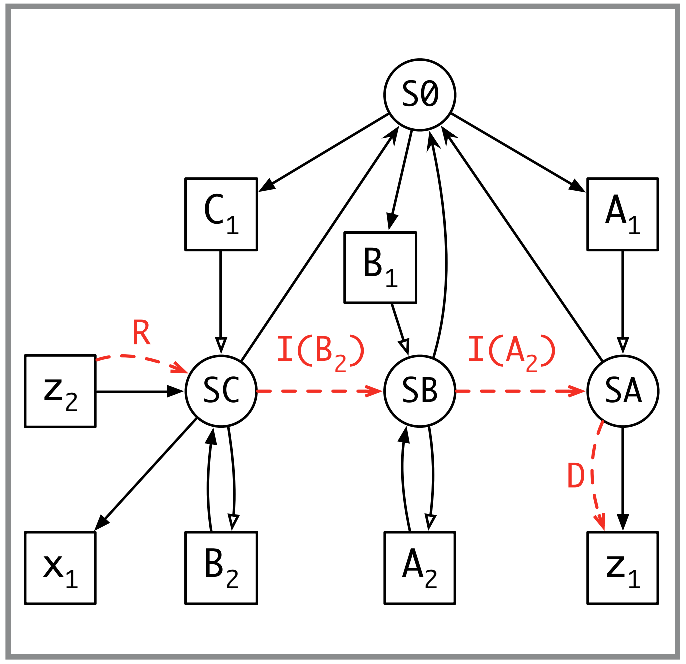


A standard formalism

Scope Graphs

Supports

Resolution

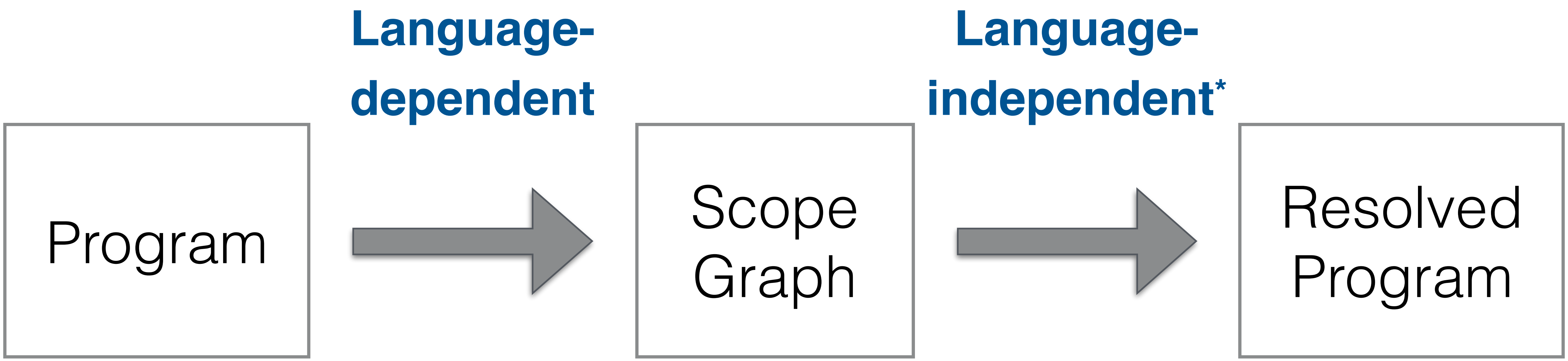
α-equivalence

IDE Navigation

Refactoring tools

Reasoning tools

Resolution Scheme



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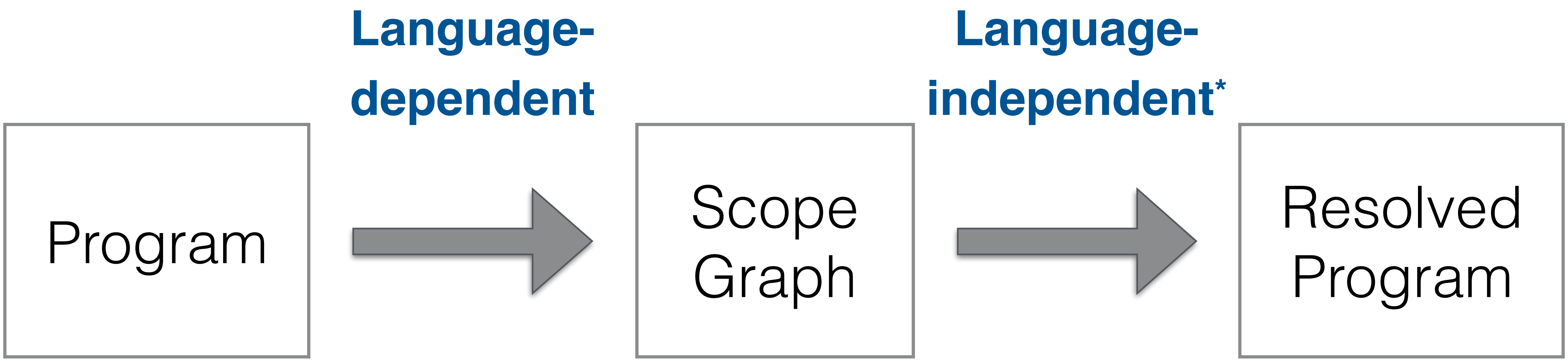

Resolution of a reference in a scope graph:

Building a **path**
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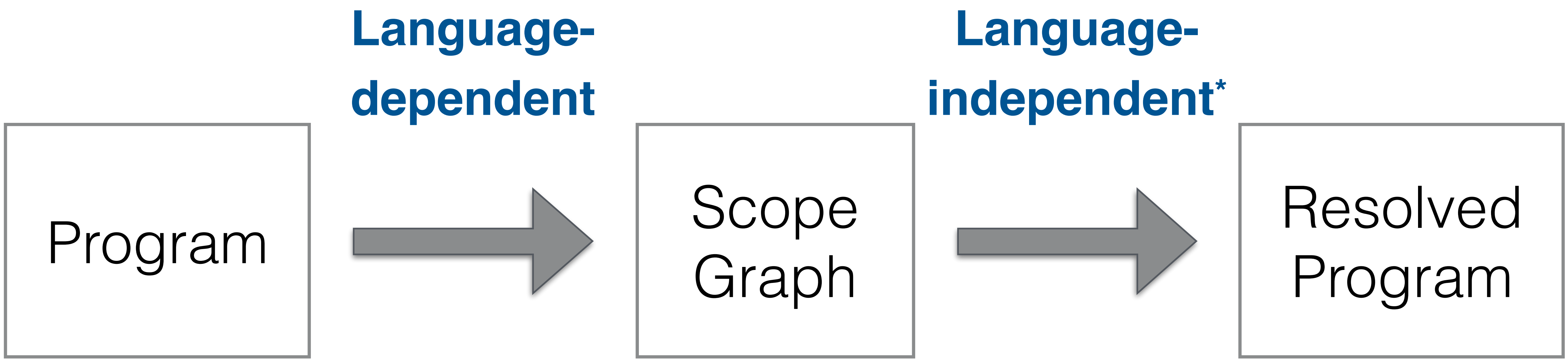

Resolution of a reference in a scope graph:

Building a **path**
from a **reference** node
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following path construction **rules**

*Parameterized by notions of path **well-formedness**
and **ordering**

Scope Graphs by Example

Simple Scopes

```
def y  = x  + 1
def x  = 5
```

Simple Scopes

$$\texttt{def } y_1 = x_2 + 1$$
$$\texttt{def } x_1 = 5$$

Simple Scopes

```
def y₁ = x₂ + 1
def x₁ = 5
```

Simple Scopes

S0

$\texttt{def}\ y_1 = x_2 + 1$

$\texttt{def}\ x_1 = 5$

Simple Scopes

S0

def $y_1 = x_2 + 1$

def $x_1 = 5$

S0

S Scope

Simple Scopes

S0

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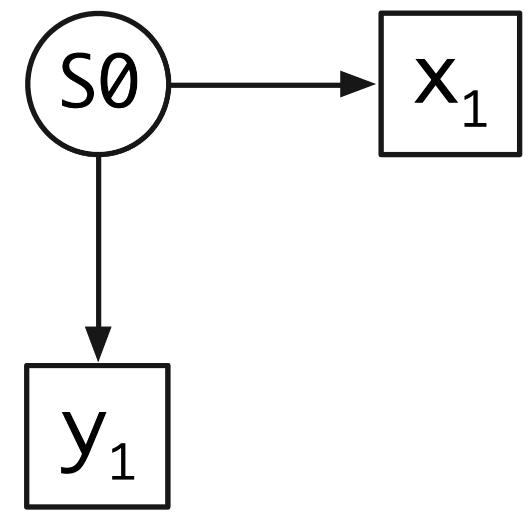

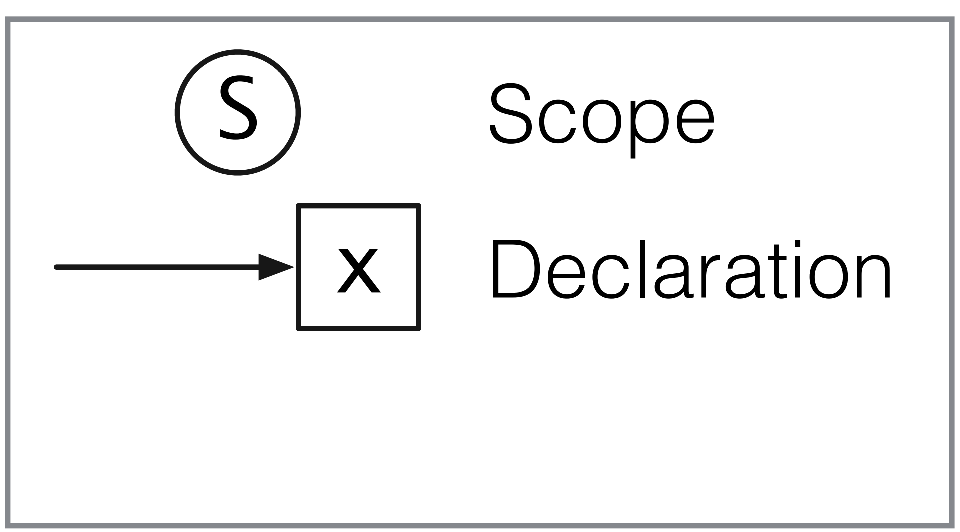

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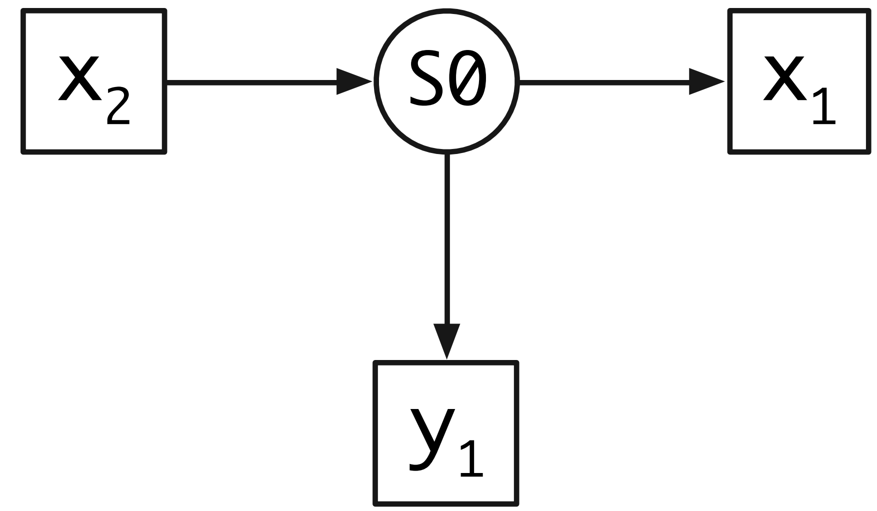

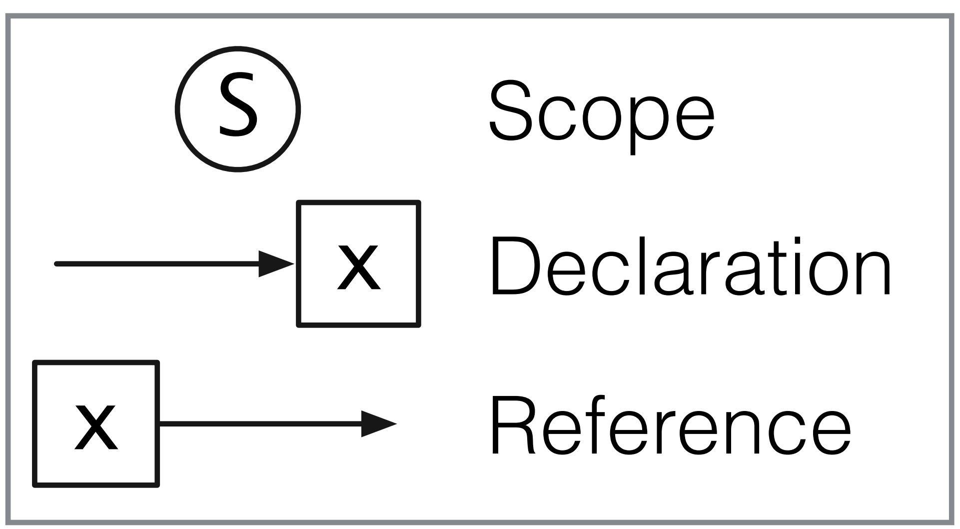

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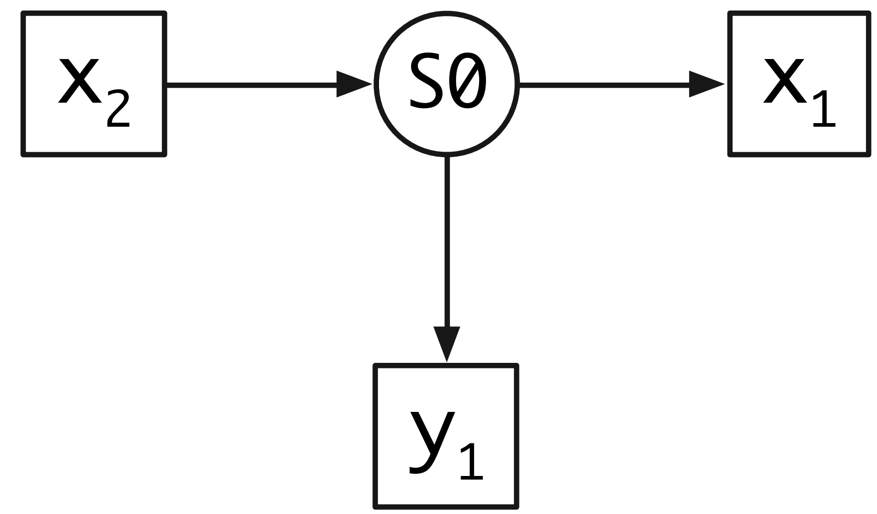

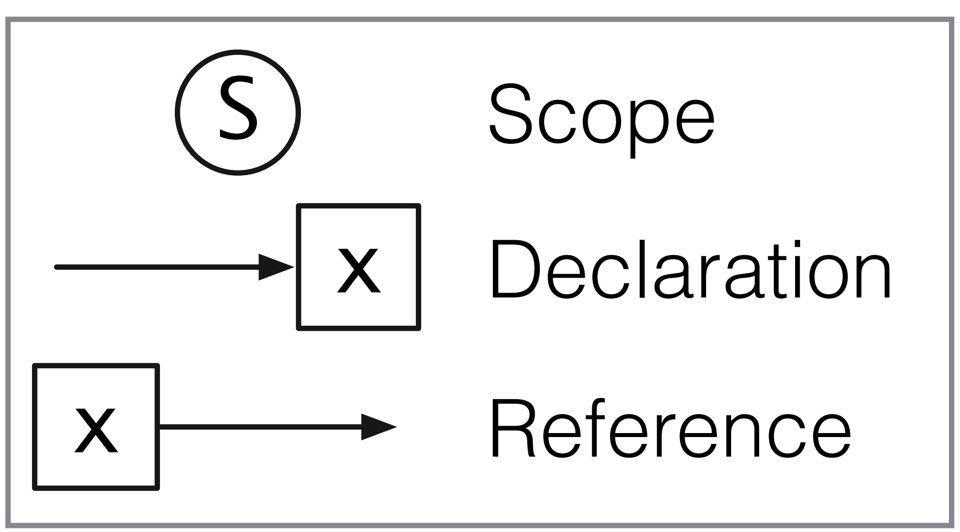

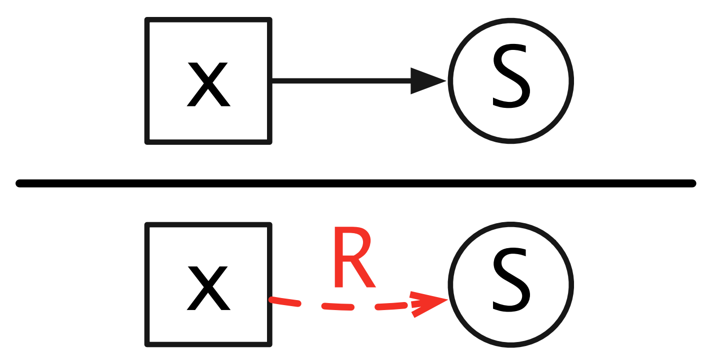

Reference Step

Simple Scopes

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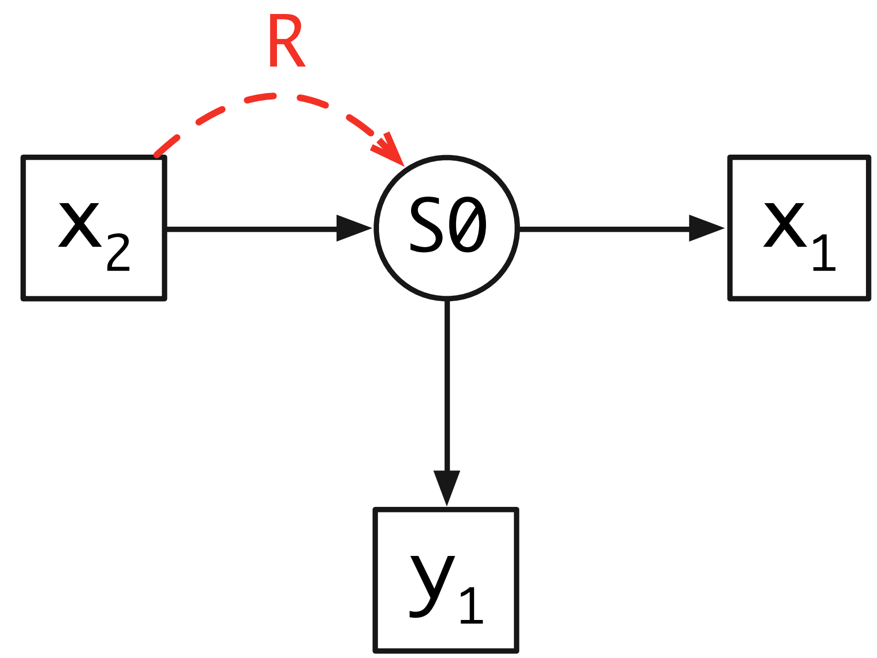

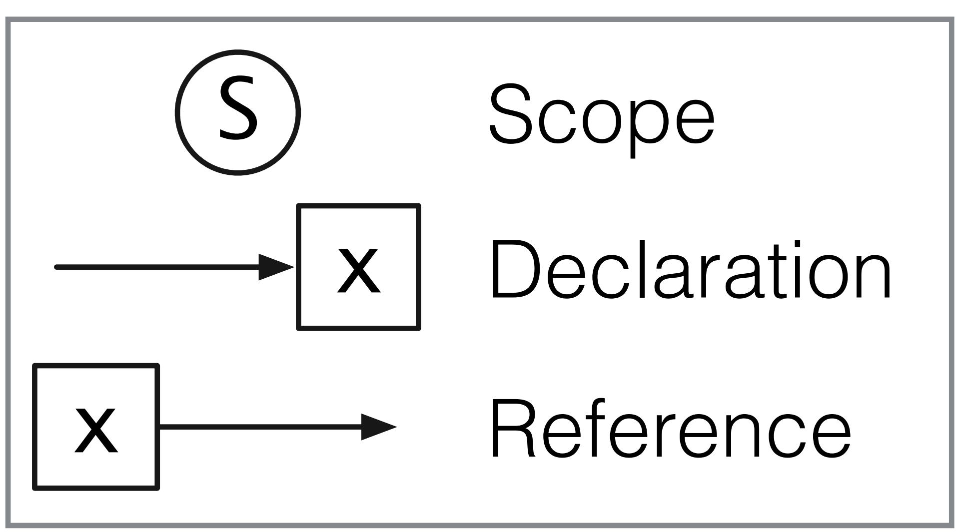

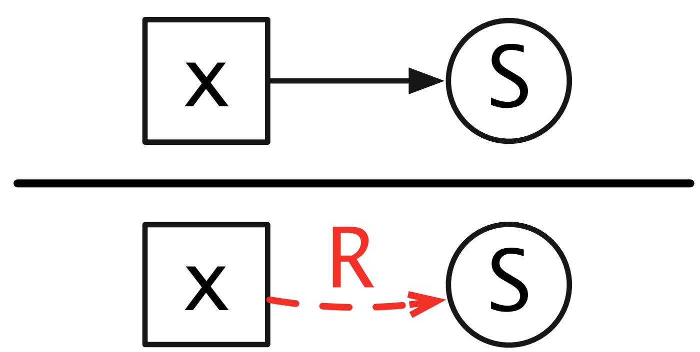

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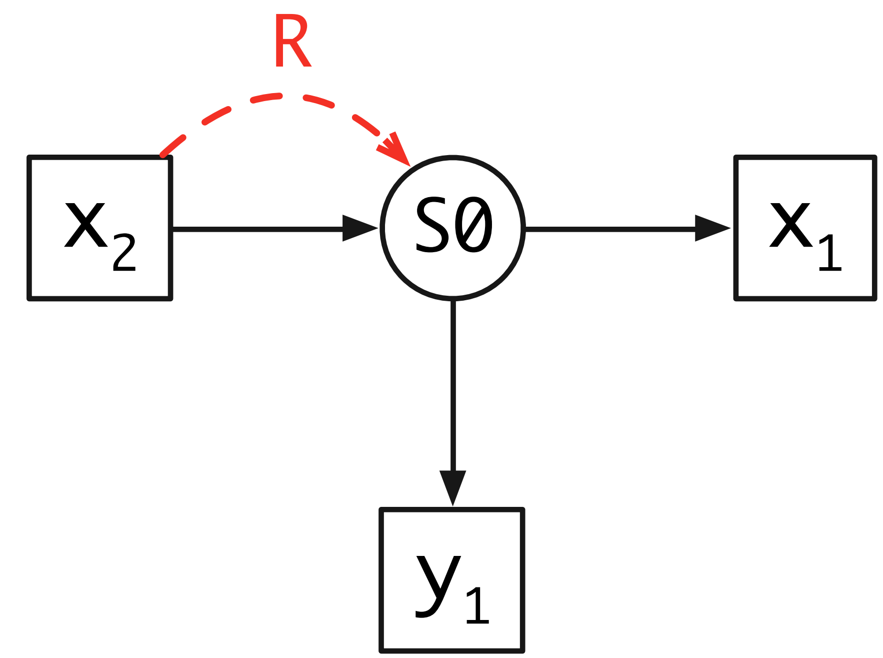

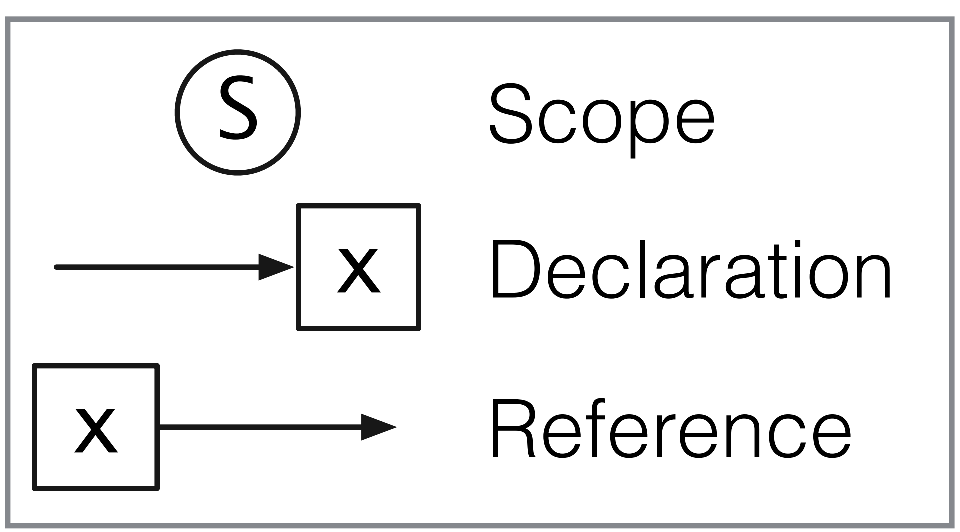

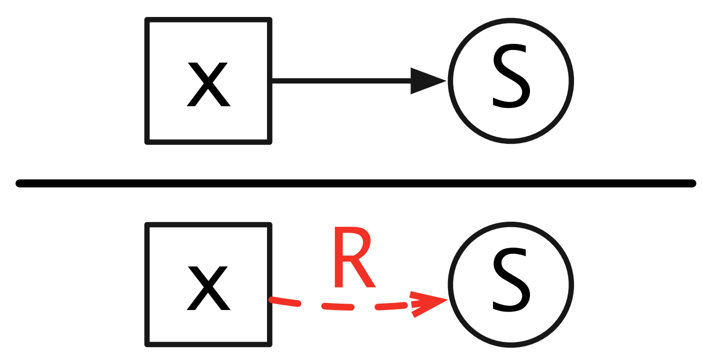

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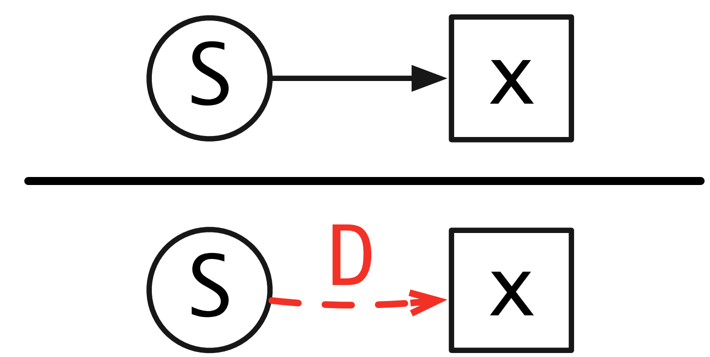

Declaration Step

Simple Scopes

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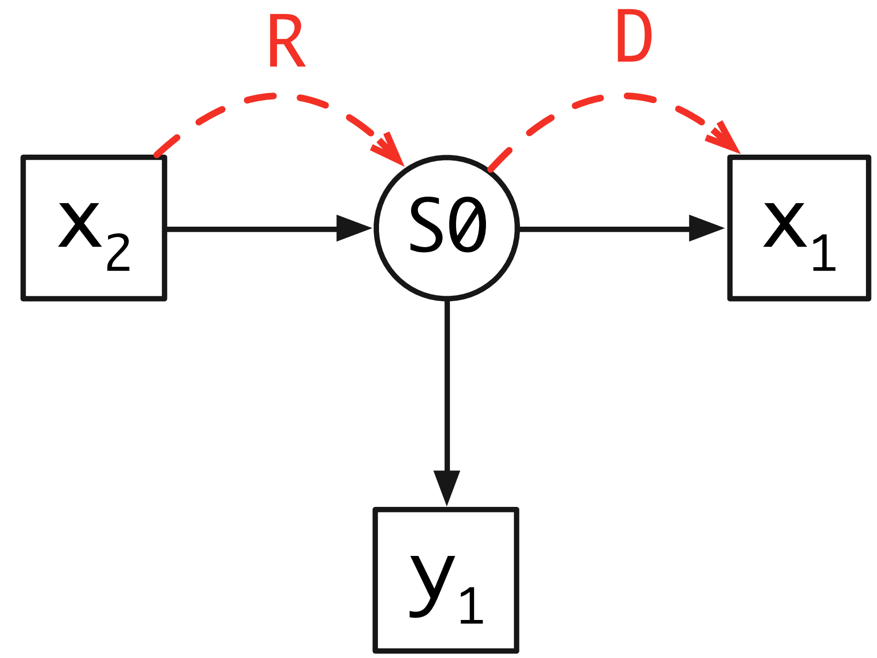
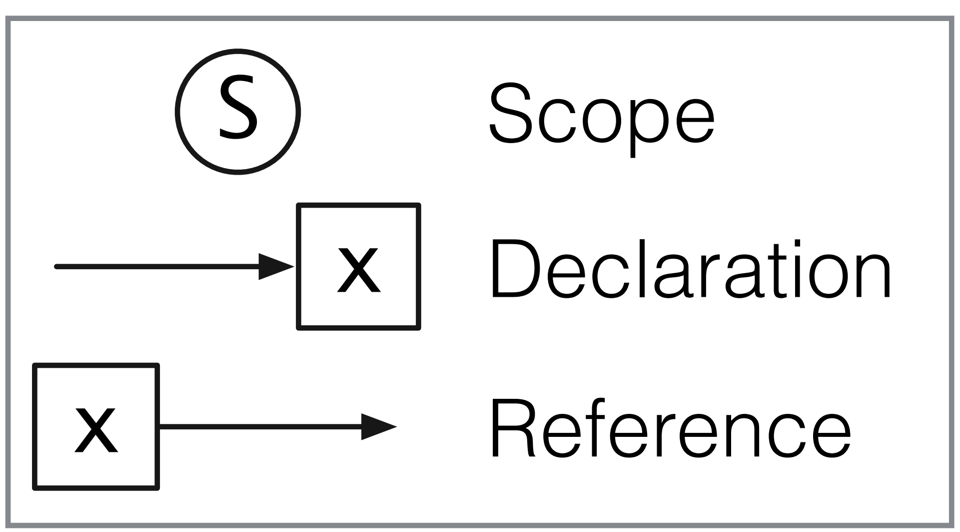
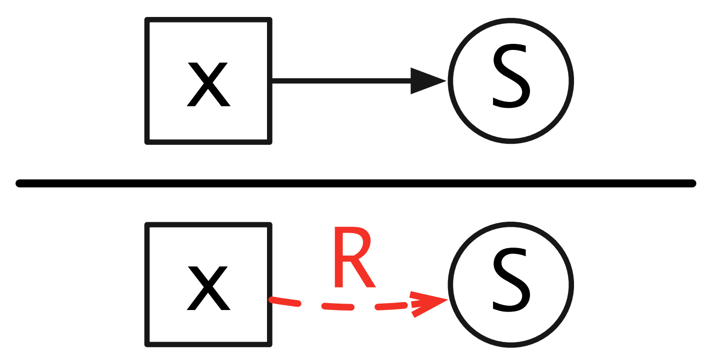
def $y_1 = x_2 + 1$

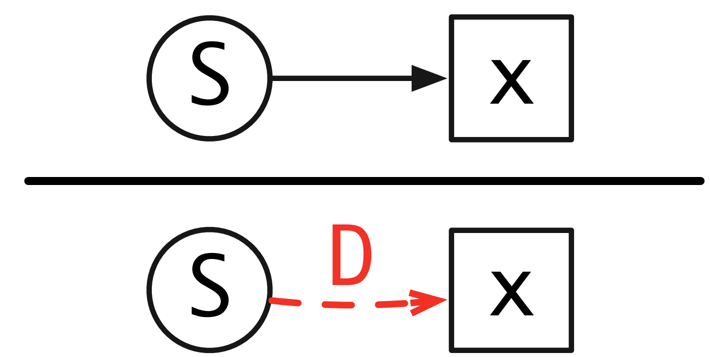
def $x_1 = 5$







Reference Step



Declaration Step

Ambiguous Resolutions

S0

```
def x₁ = 5

def x₂ = 3

def z₁ = x₃ + 1
```

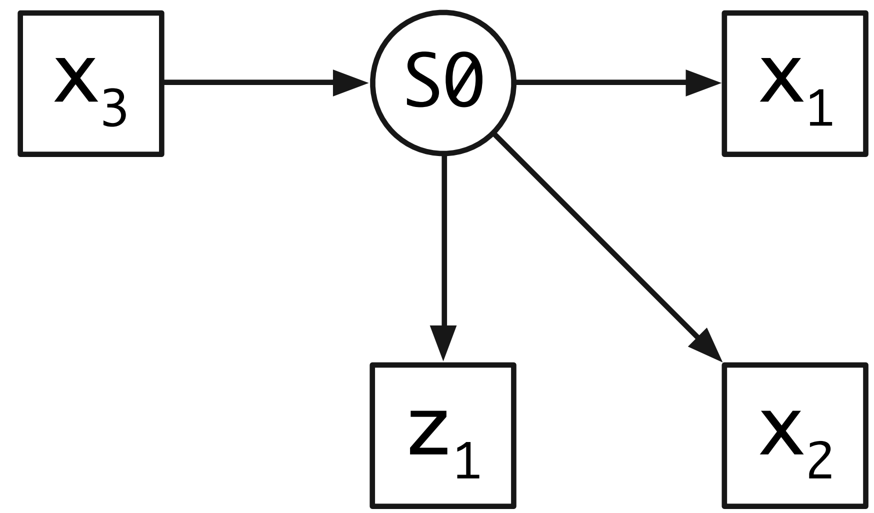
Ambiguous Resolutions

S0

$\mathtt{def}\ \boxed{x_1} = 5$

$\mathtt{def}\ \boxed{x_2} = 3$

$\mathtt{def}\ z_1 = x_3 + 1$



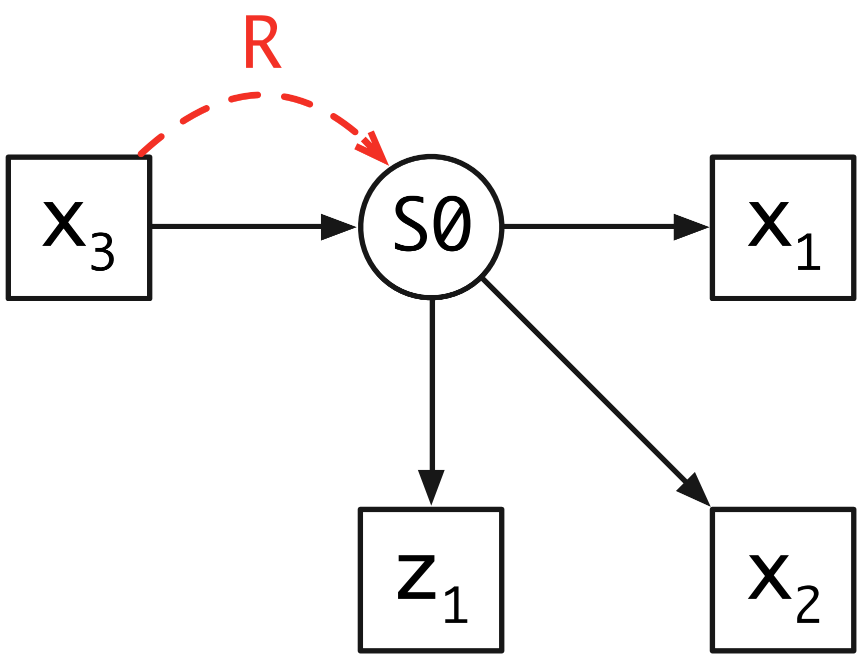
Ambiguous Resolutions

S0

$\texttt{def}\ \boxed{x_1} = 5$

$\texttt{def}\ \boxed{x_2} = 3$

$\texttt{def}\ z_1 = x_3 + 1$



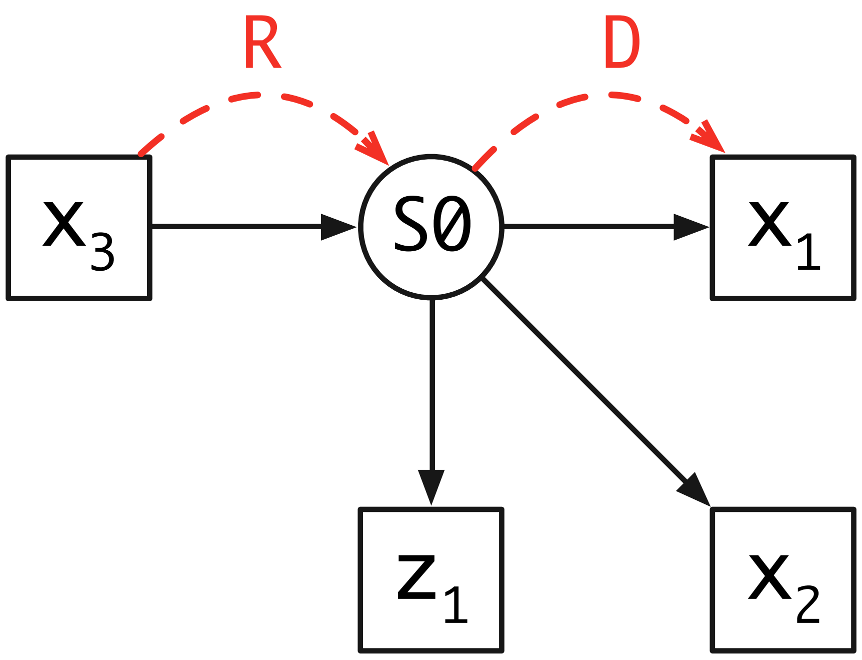
Ambiguous Resolutions

S0

```
def x₁ = 5

def x₂ = 3

def z₁ = x₃ + 1
```

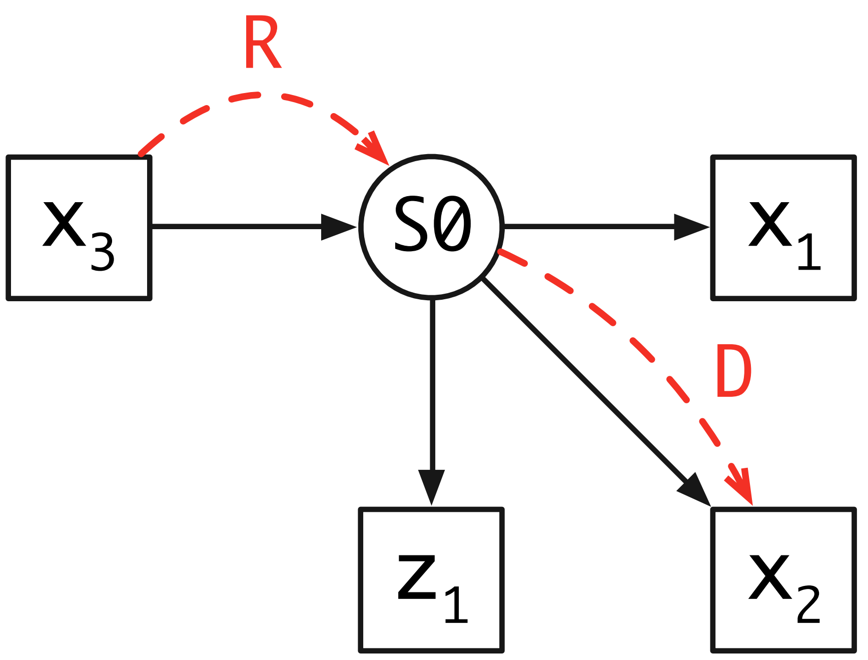


Ambiguous Resolutions

```
def x₁ = 5                    S0

def x₂ = 3

def z₁ = x₃ + 1
```



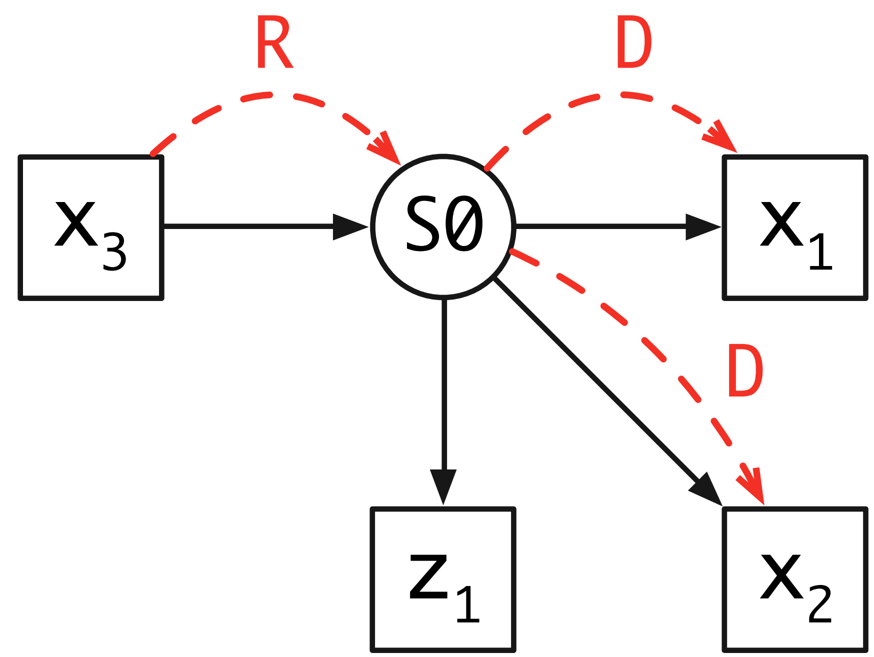
Ambiguous Resolutions

S0

$\texttt{def}\ \boxed{x_1} = 5$

$\texttt{def}\ \boxed{x_2} = 3$

$\texttt{def}\ z_1 = x_3 + 1$



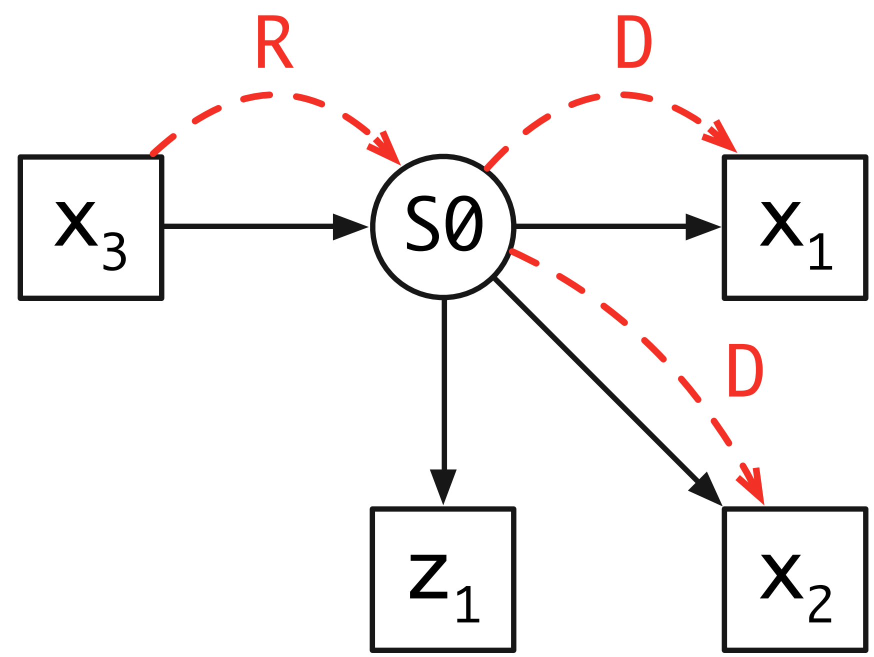
Ambiguous Resolutions

S0

def $\boxed{x_1}$ = 5

def $\boxed{x_2}$ = 3

def z_1 = x_3 + 1



match t with

| A $\boxed{x}$ | B $\boxed{x}$ => …

Lexical Scoping

```
def x1 = z2 5

def z1 =
  fun y1 {
    x2 + y2
  }
```

Lexical Scoping

S0

$\texttt{def}\ x_1 = z_2\ 5$

$\texttt{def}\ z_1 =$

$\quad \texttt{fun}\ y_1\ \{$

$\qquad x_2 + y_2$

$\quad \}$

Lexical Scoping

S0

def x_1 = z_2 5

def z_1 =

fun y_1 {   S1

  x_2 + y_2

}

Lexical Scoping

S0

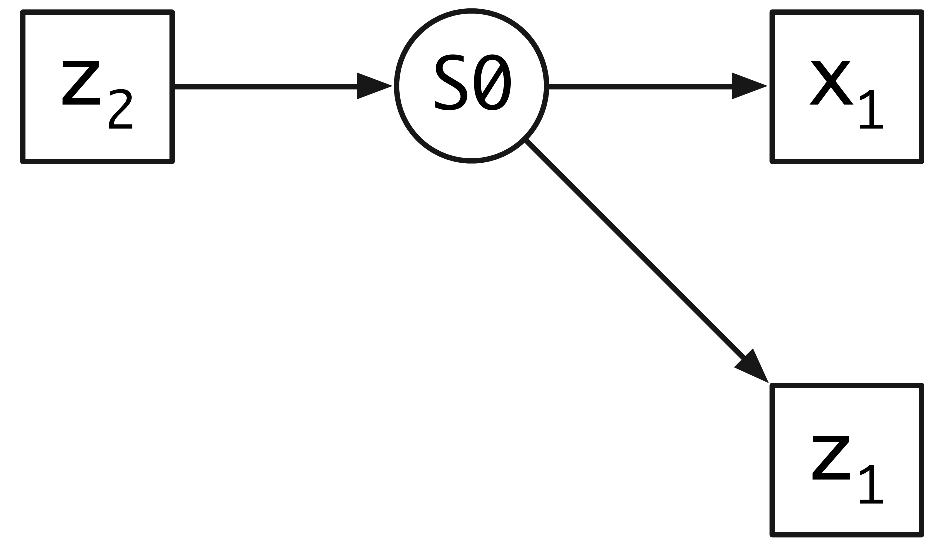
$\mathtt{def}\ x_1 = z_2\ 5$

$\mathtt{def}\ z_1 =$

$\quad \mathtt{fun}\ y_1\ \{$ S1

$\qquad x_2 + y_2$

$\quad \}$



Lexical Scoping

S0

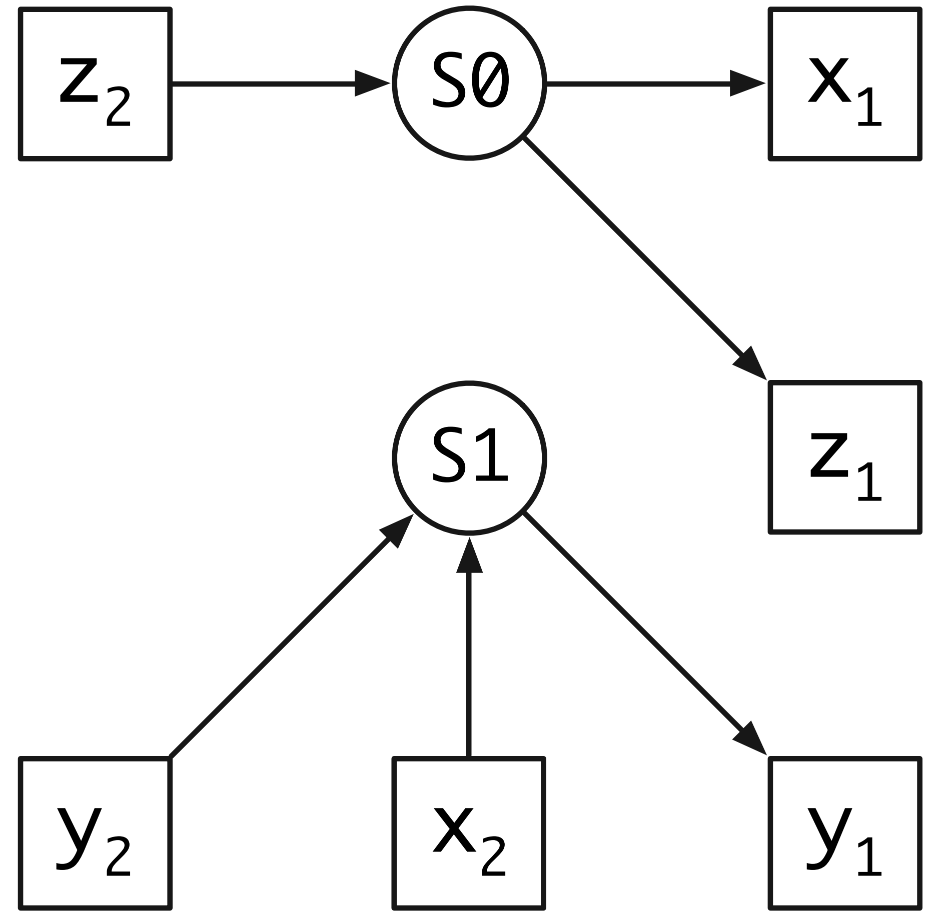
def x_1 = z_2 5

def z_1 =

fun y_1 { S1

x_2 + y_2

}

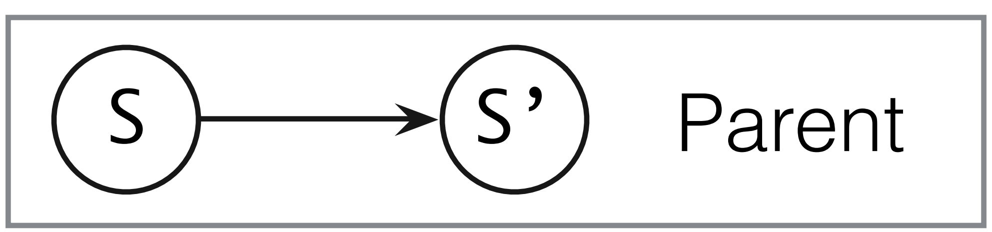


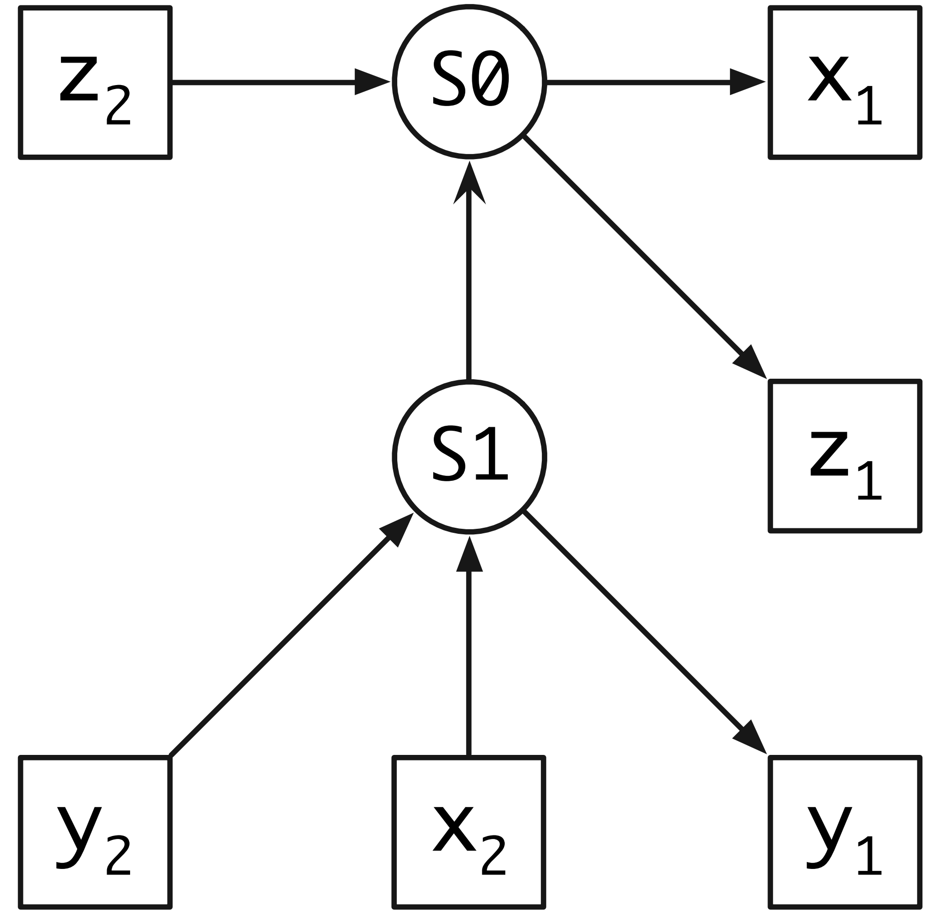
Lexical Scoping

S0

def x_1 = z_2 5

def z_1 =
  fun y_1 {   S1
    x_2 + y_2
  }





Lexical Scoping

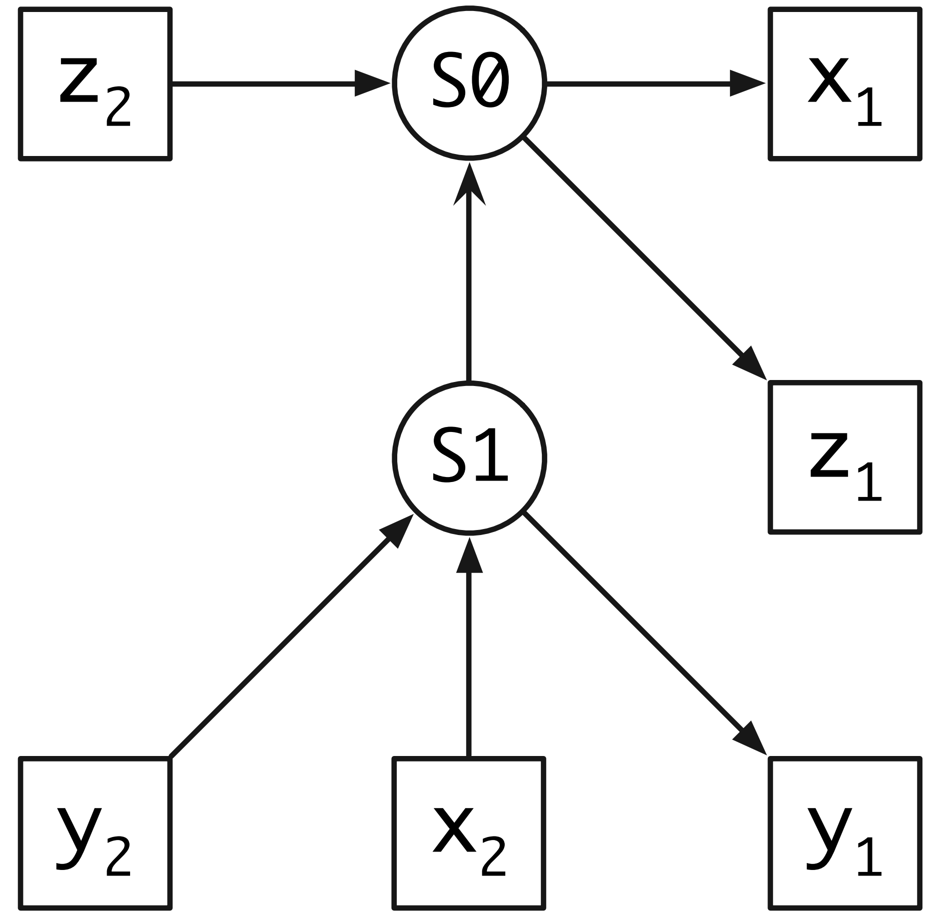
S0

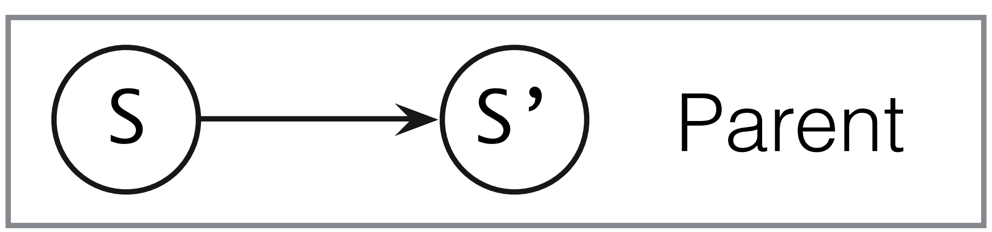
def x_1 = z_2 5

def z_1 =

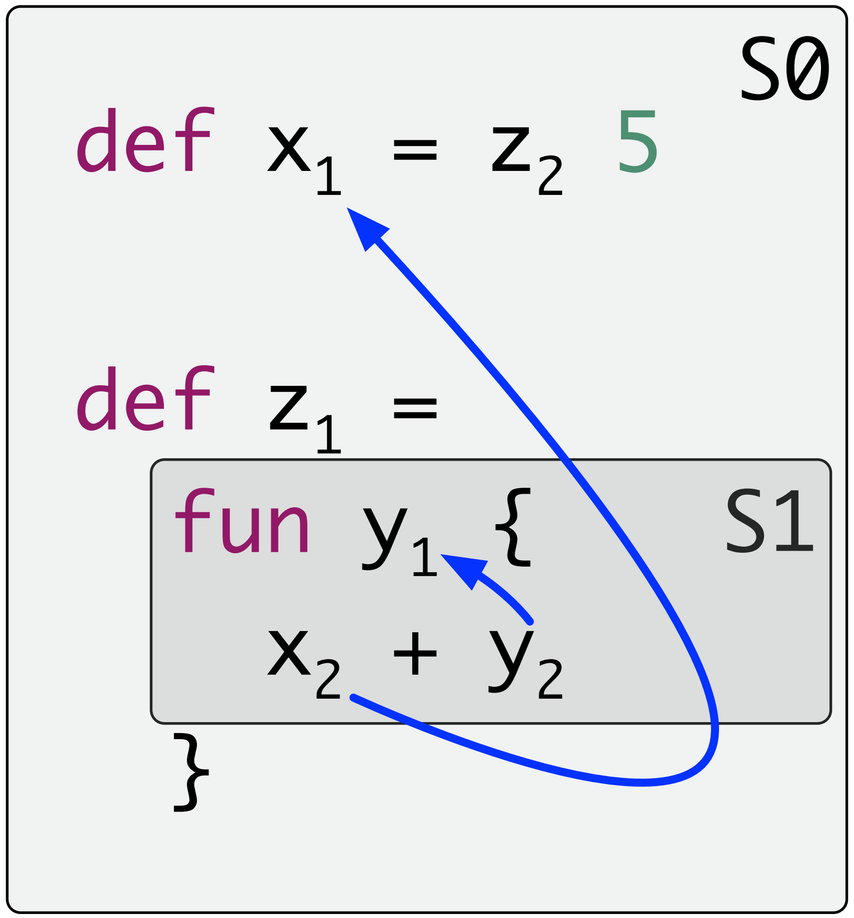
S1

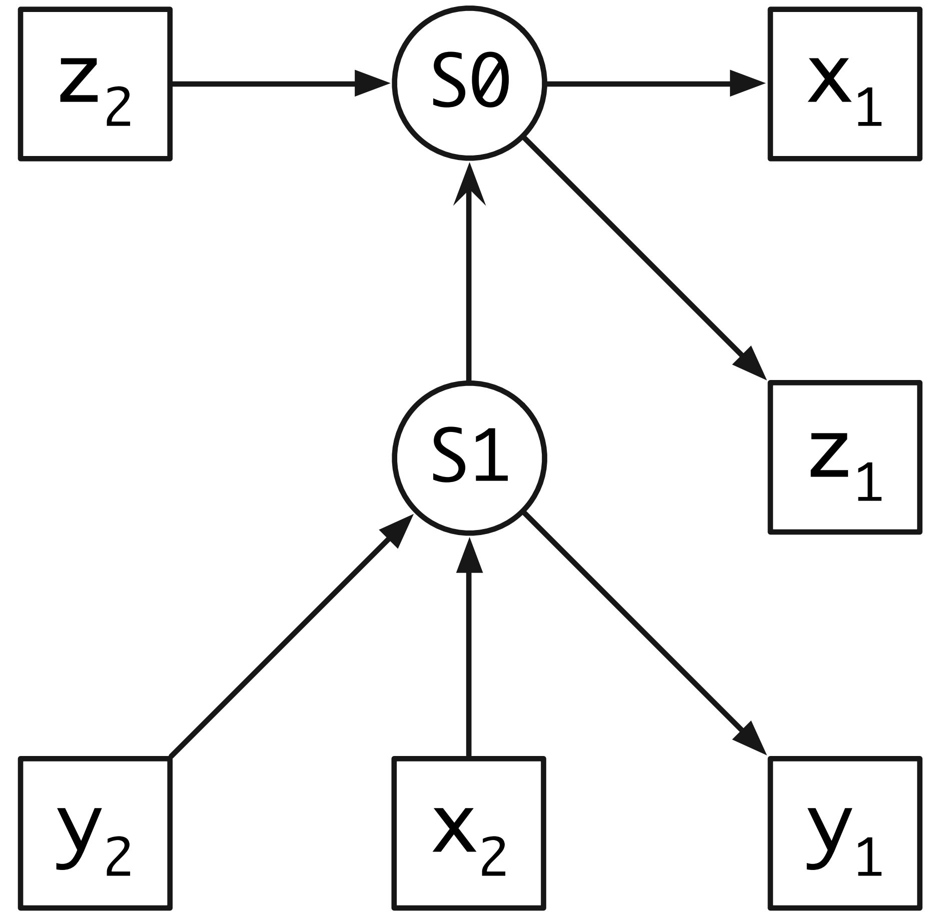
fun y_1 {
  x_2 + y_2
}

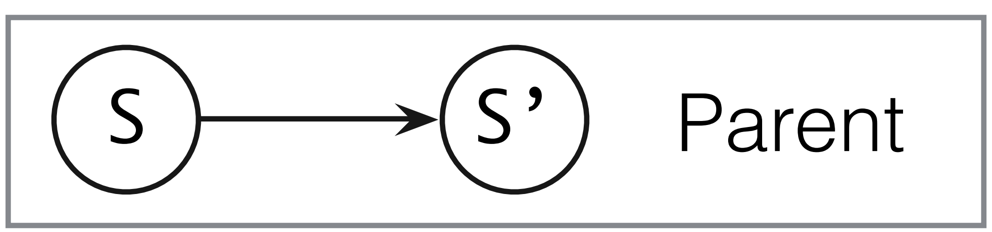




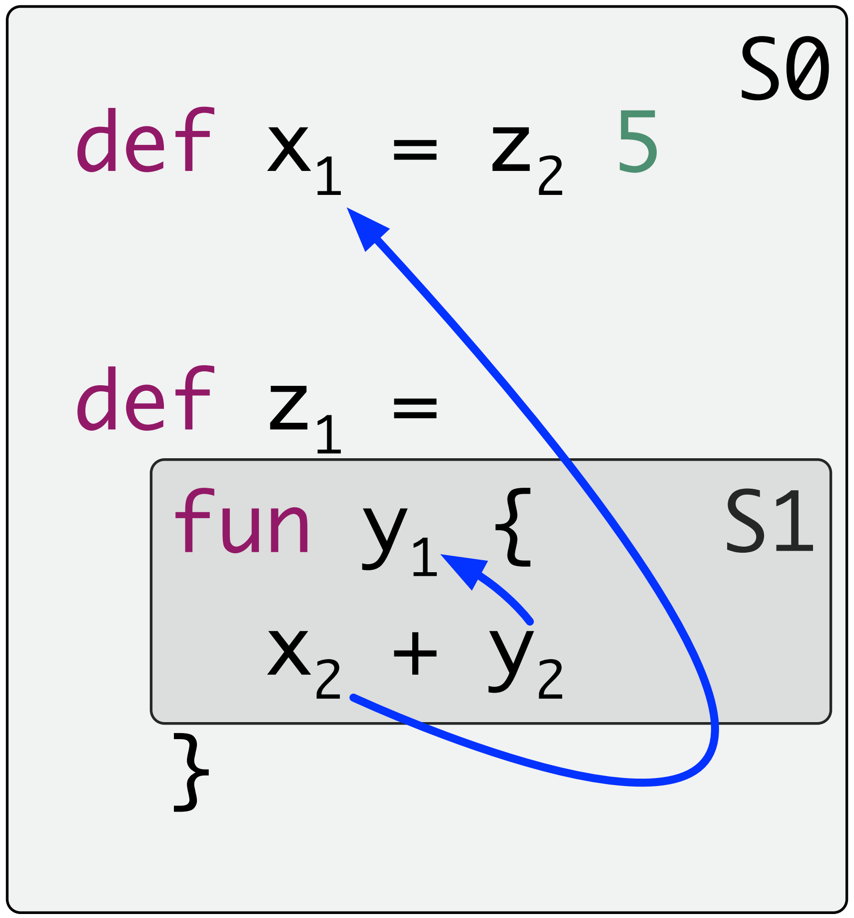
Lexical Scoping

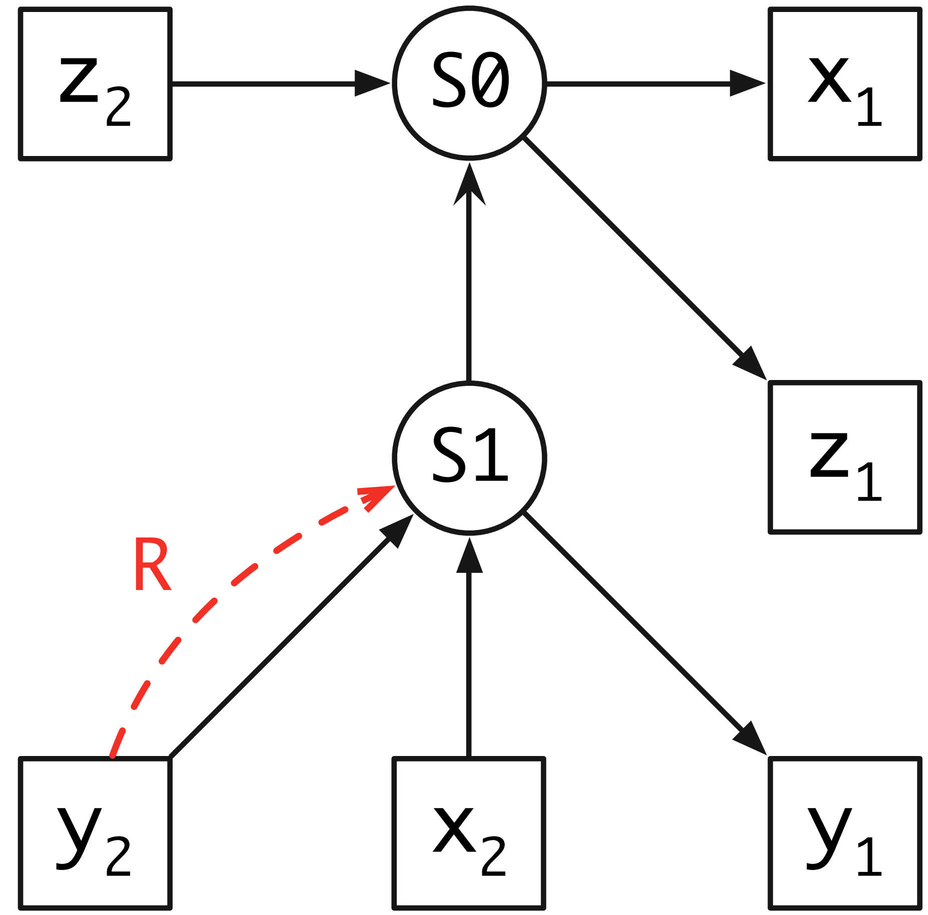


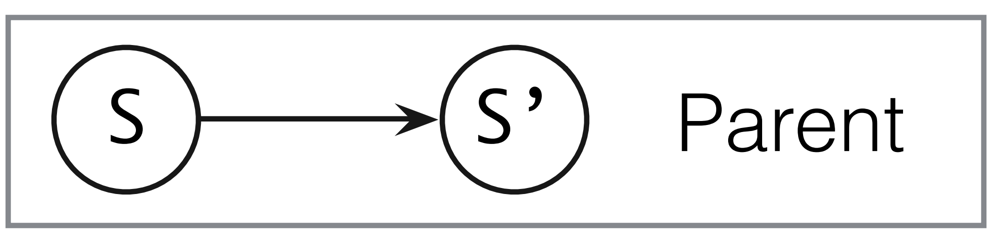




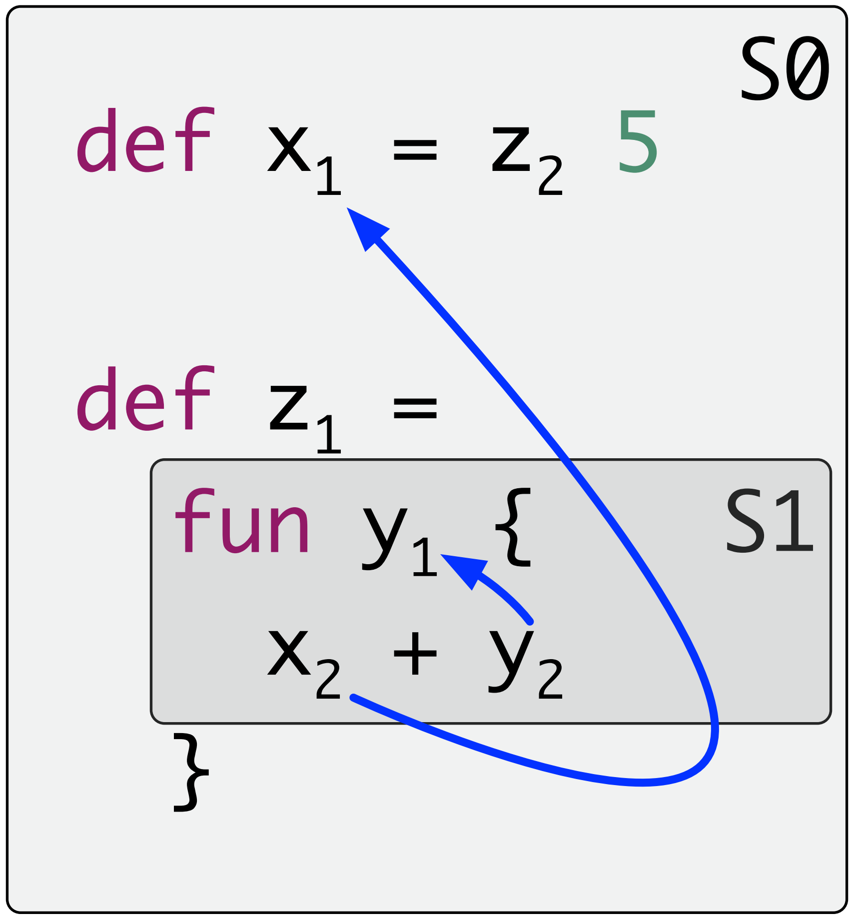
Lexical Scoping

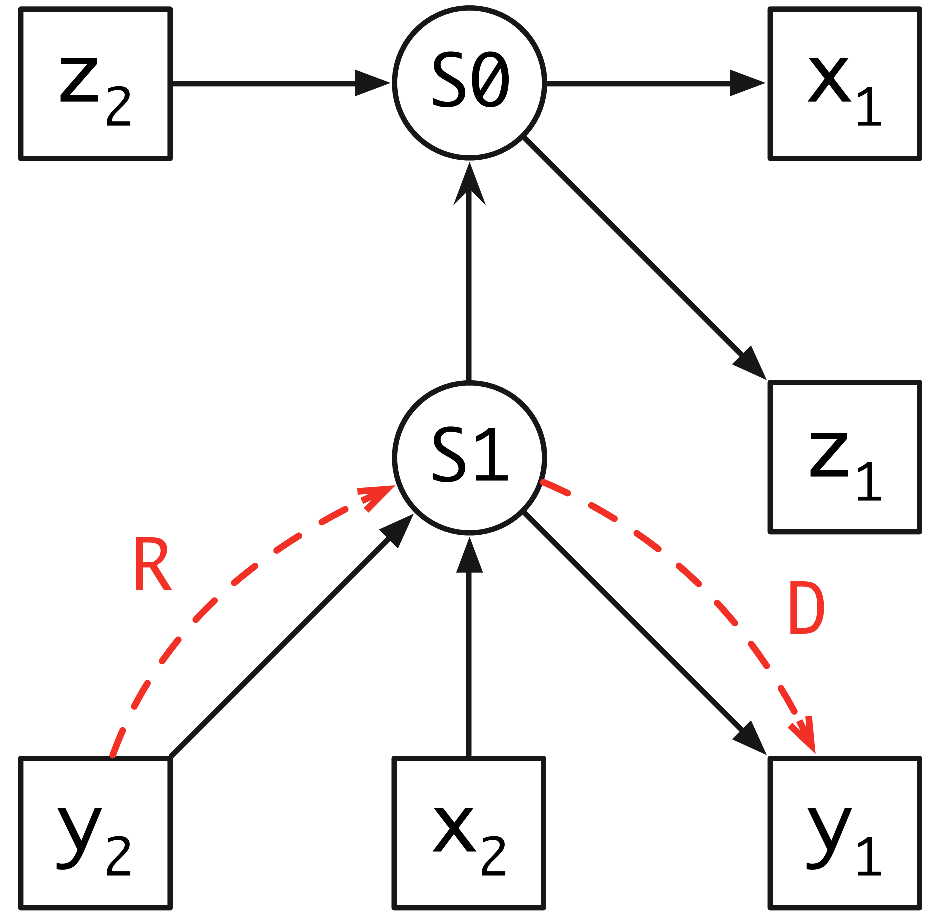


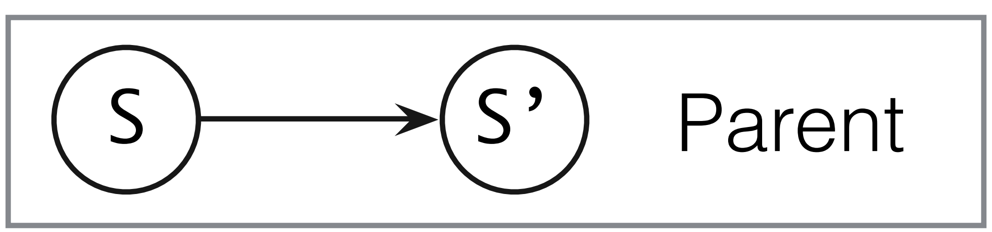




Lexical Scoping







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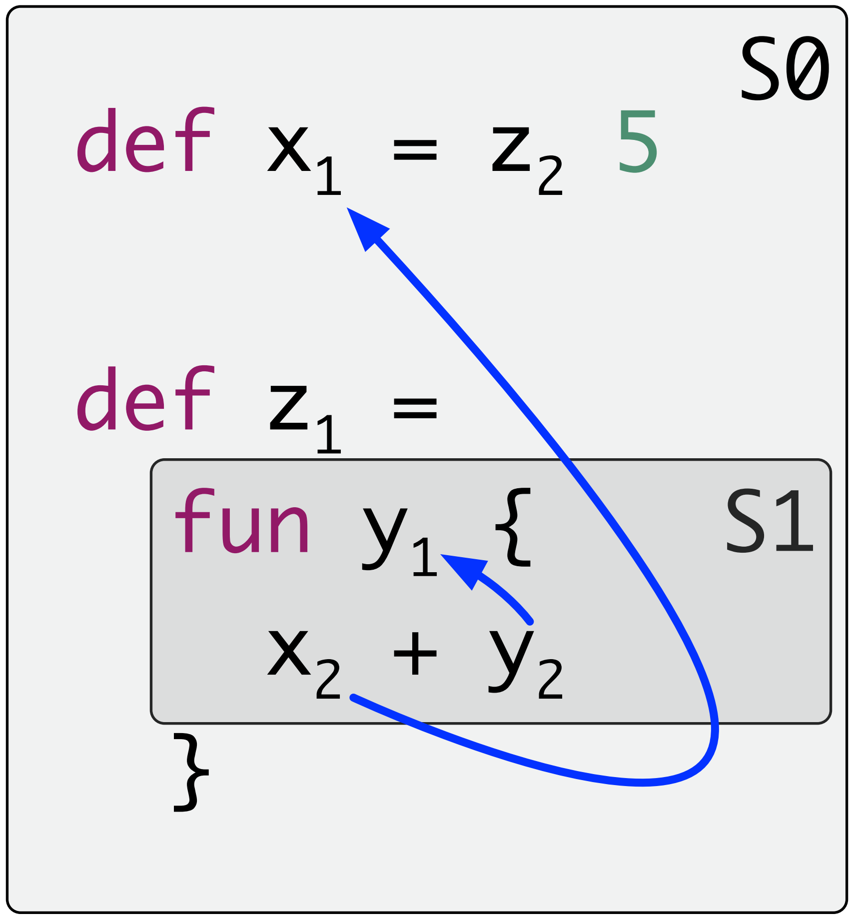
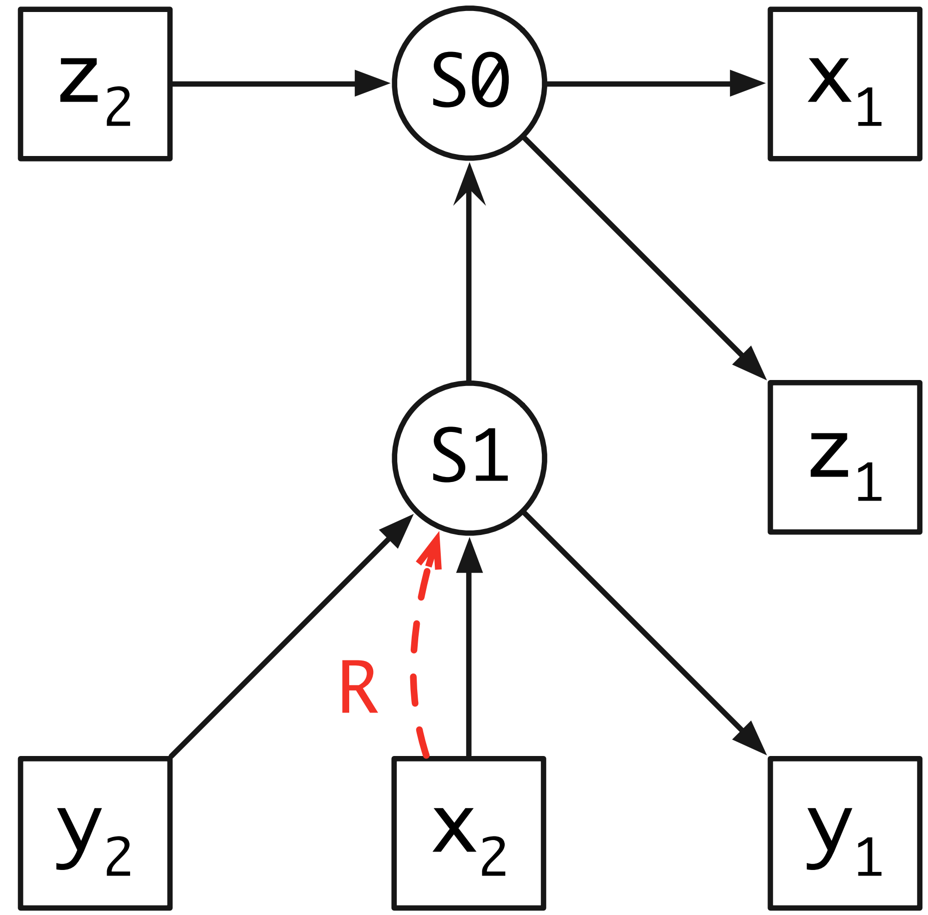
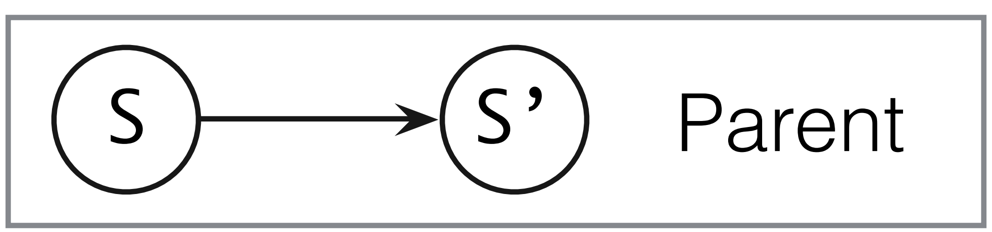
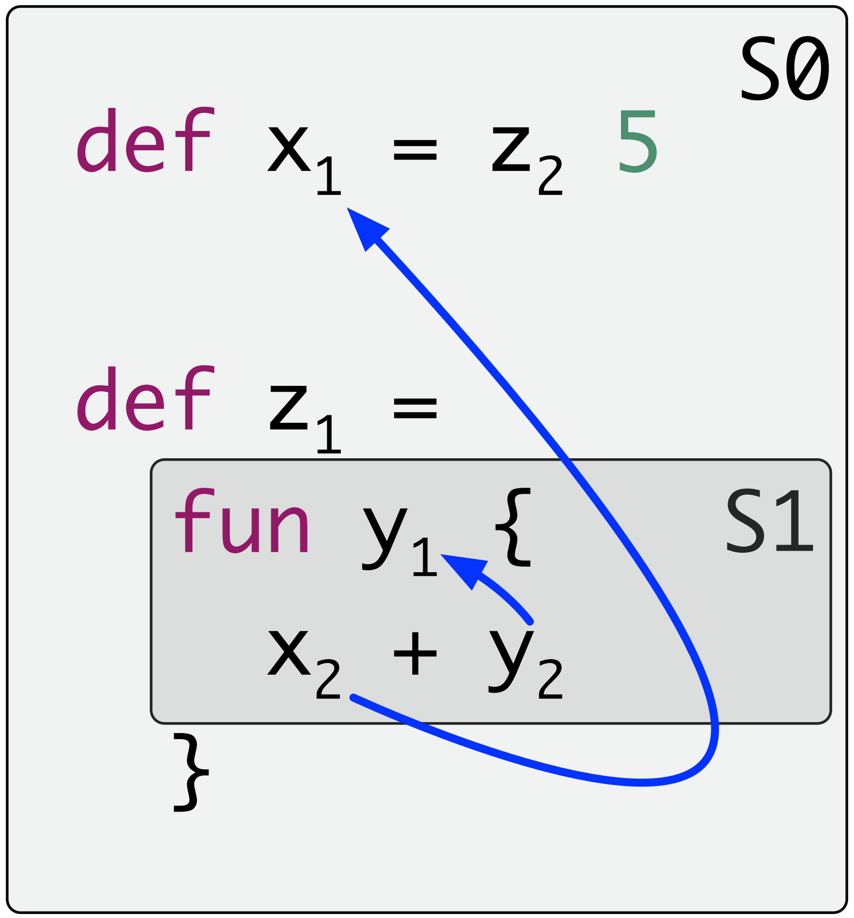
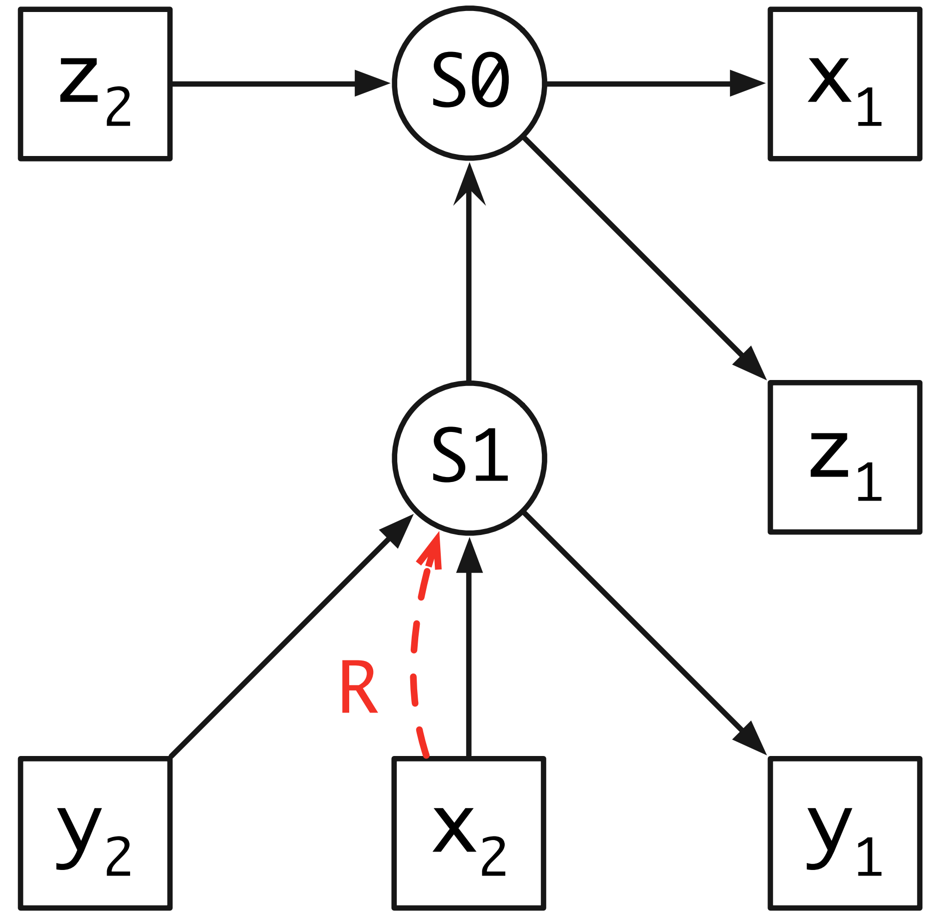


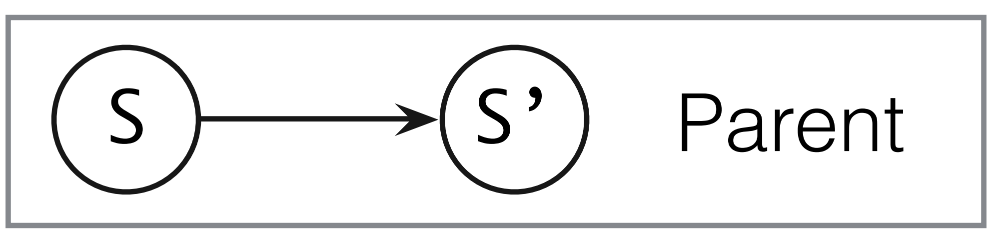


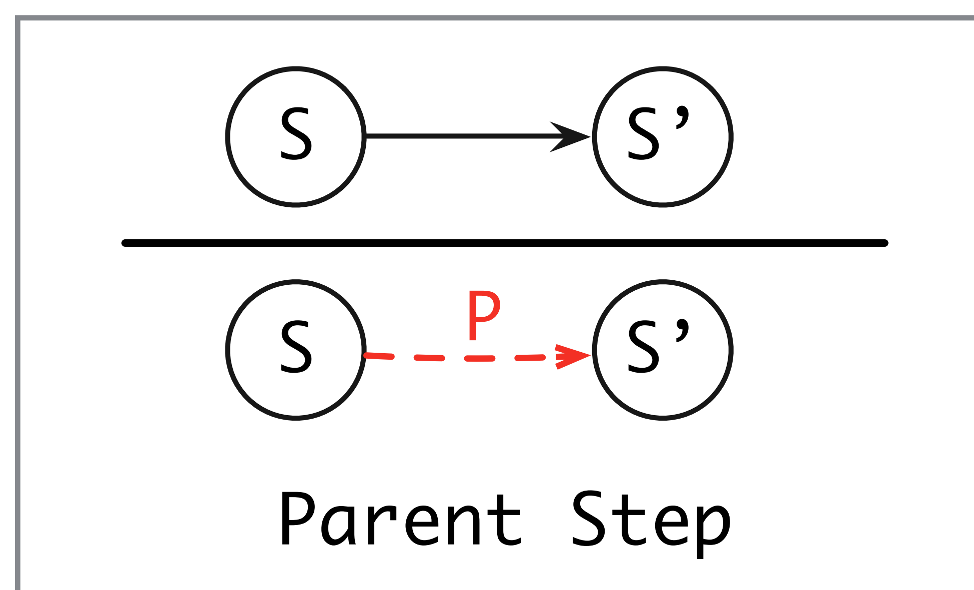


Lexical Scoping









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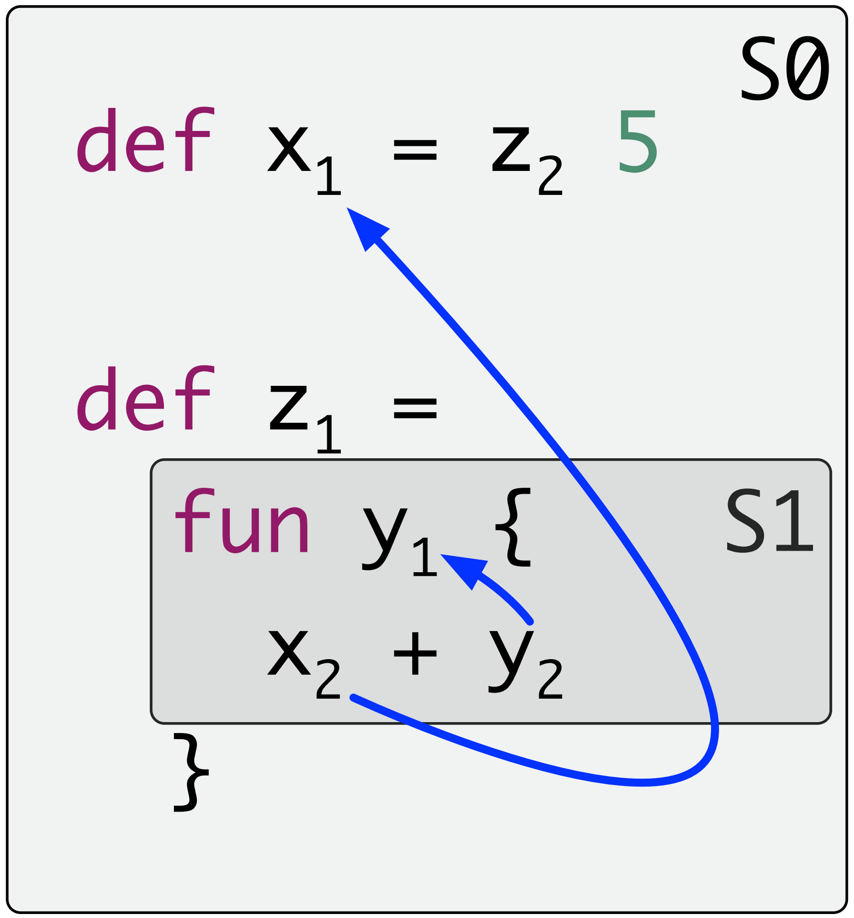

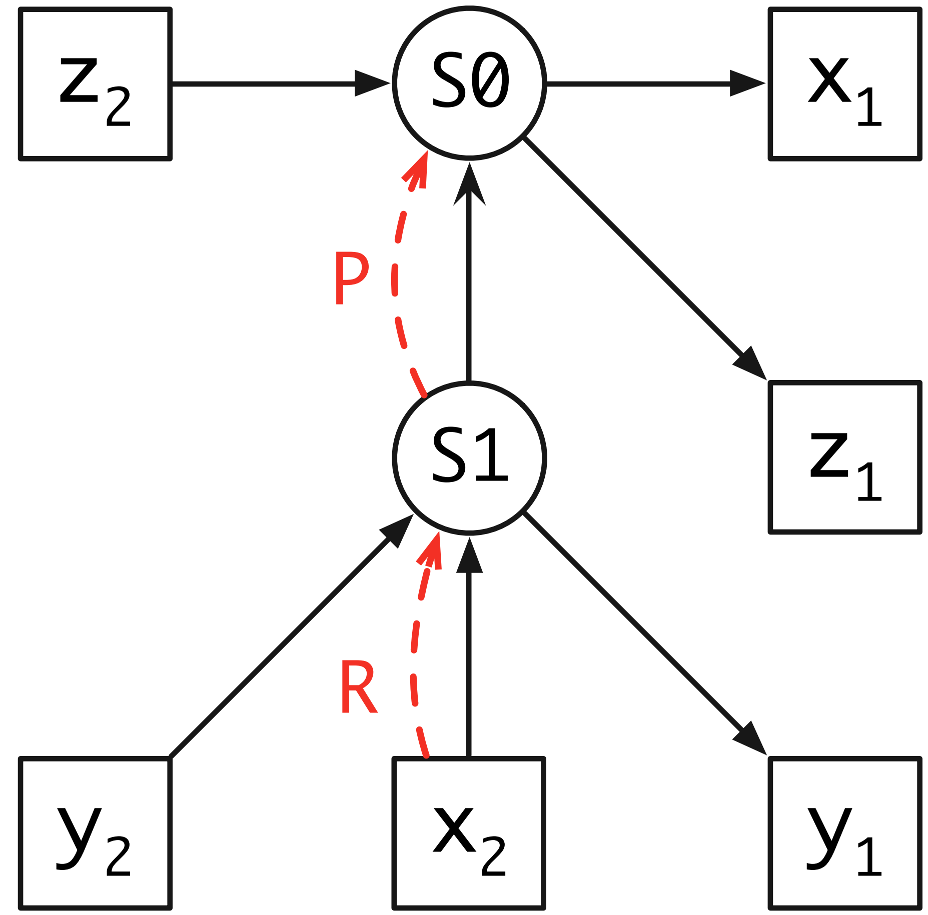

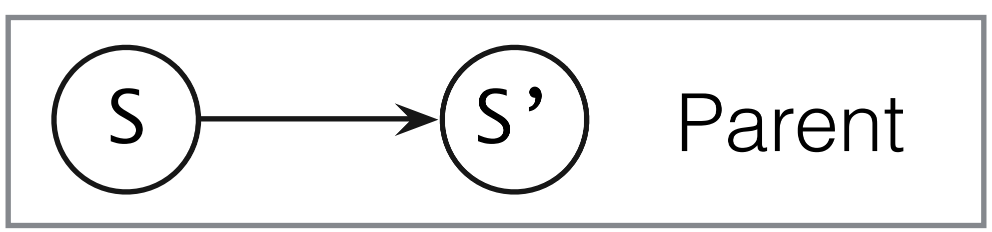

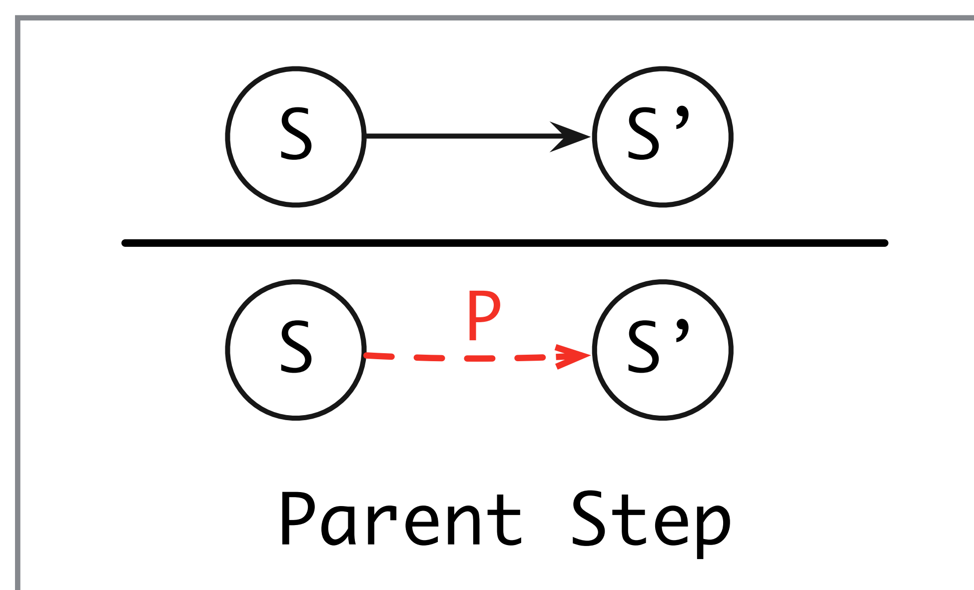

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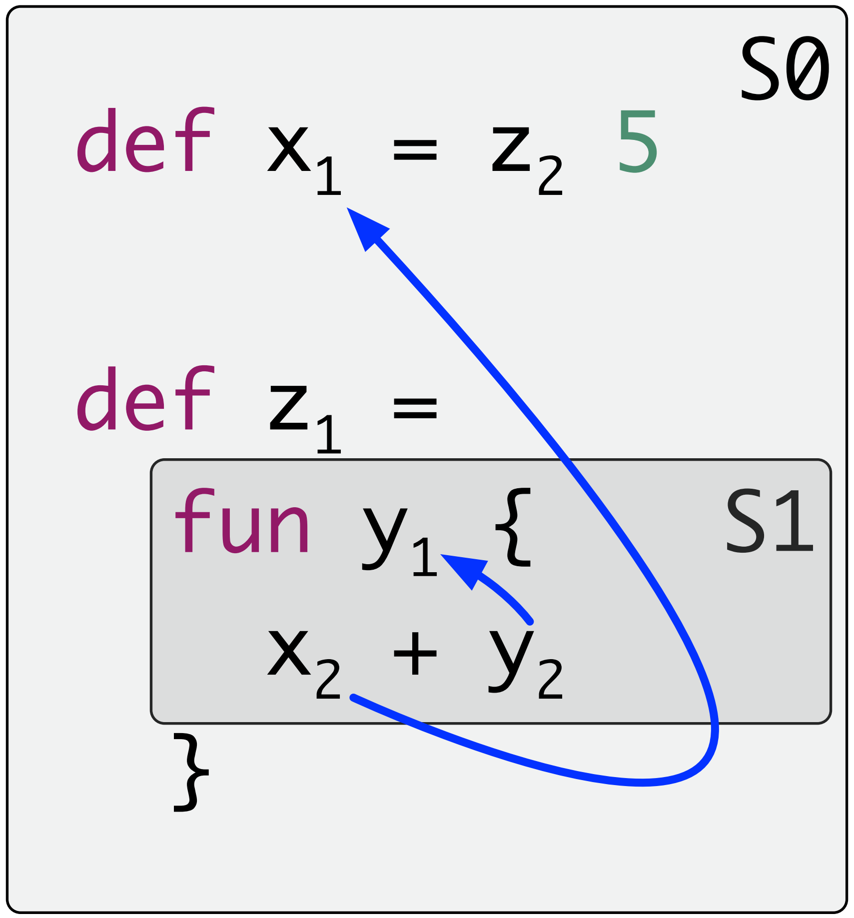
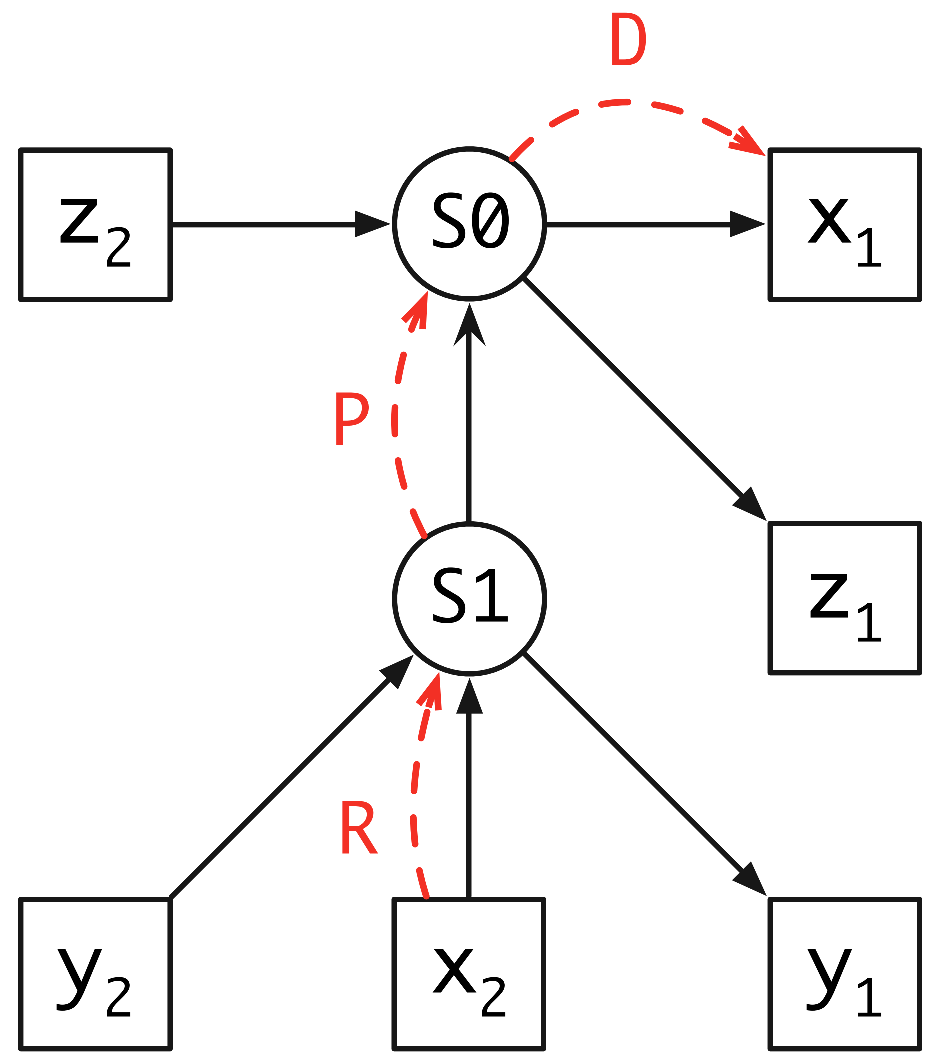
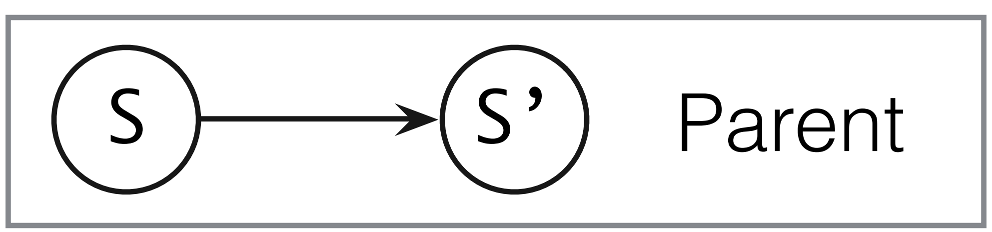
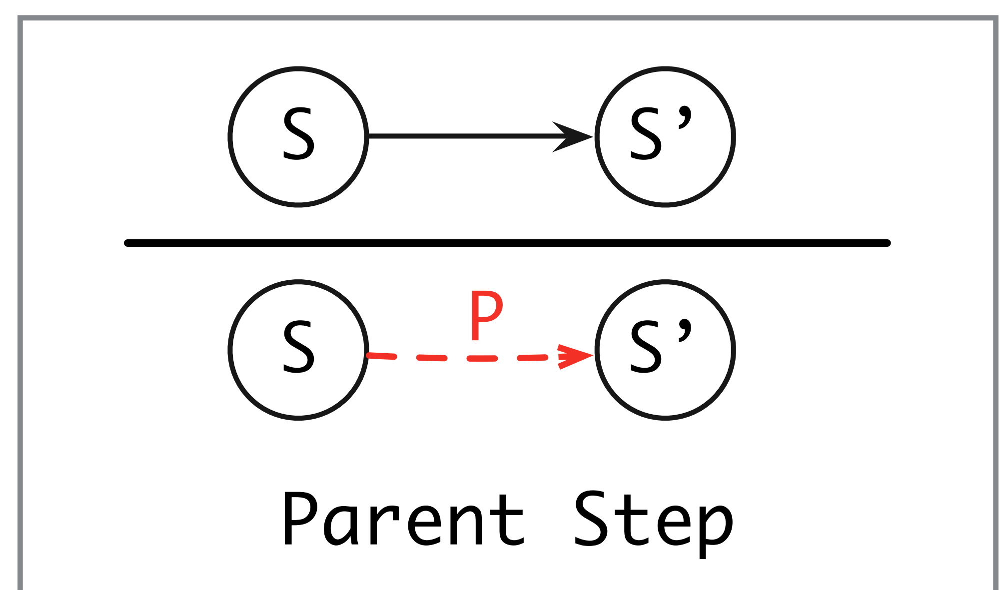
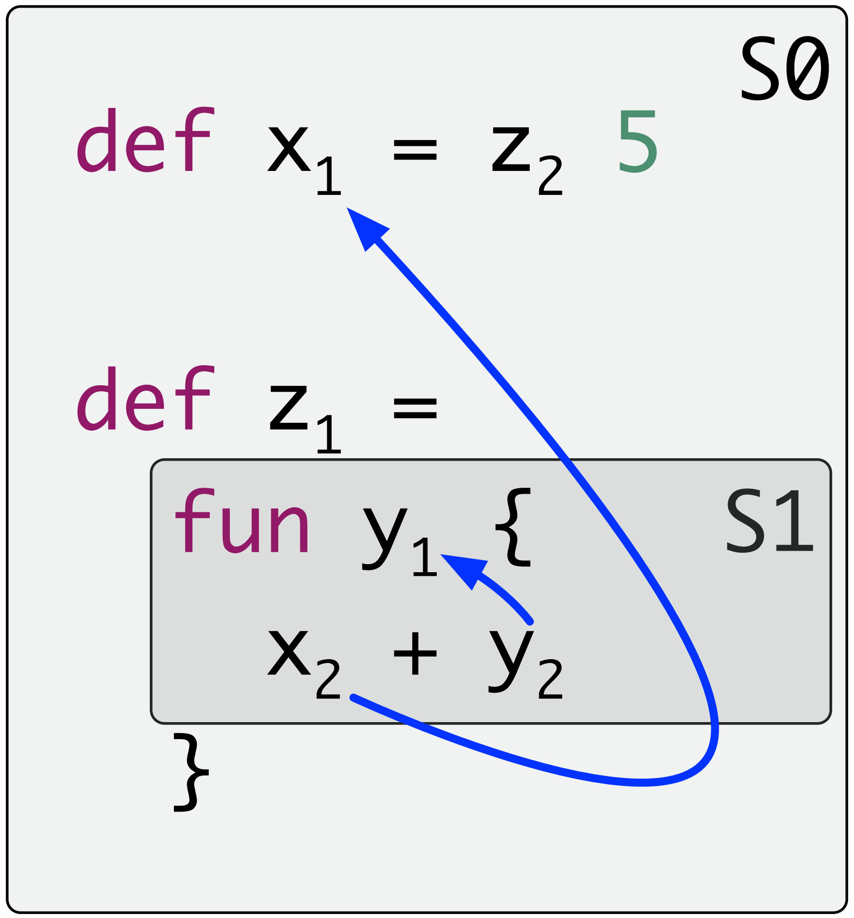


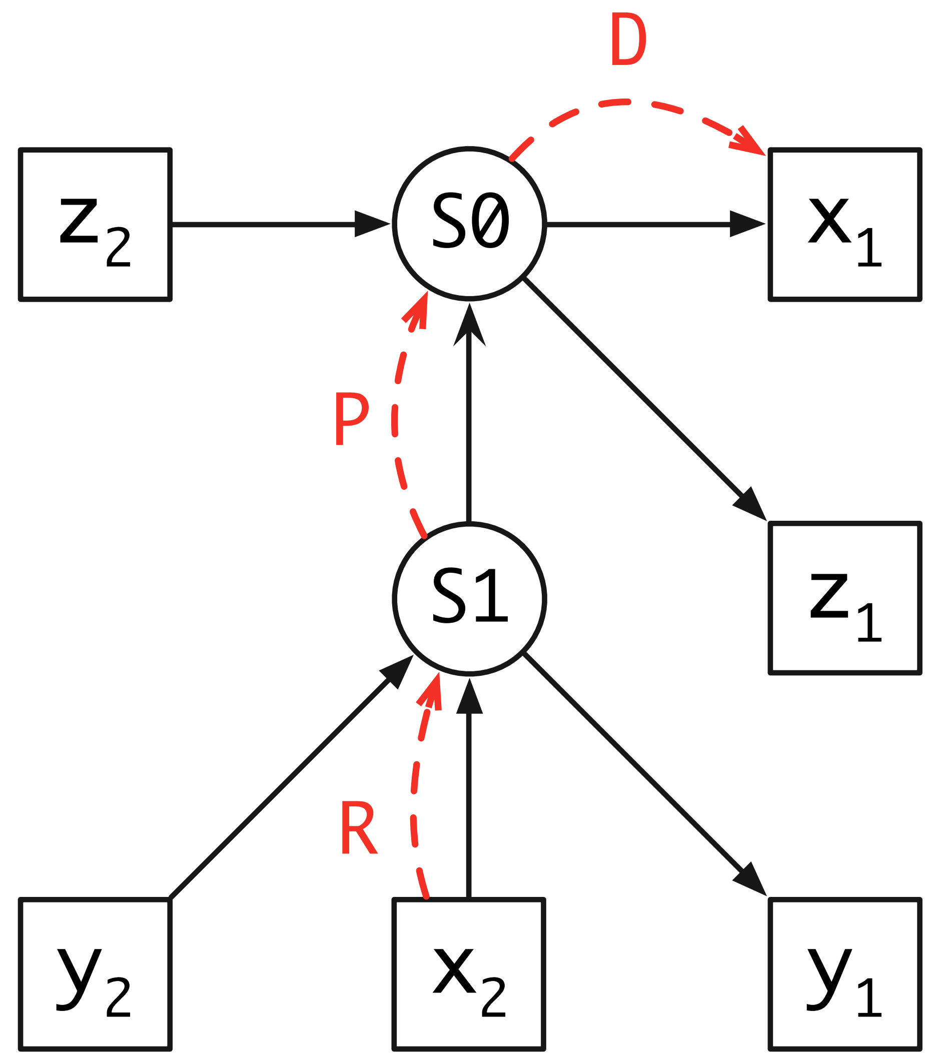


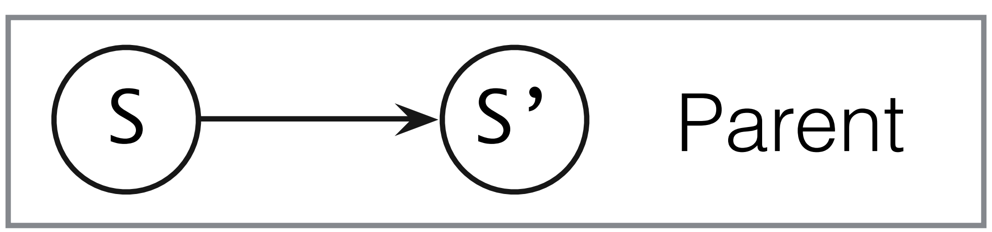


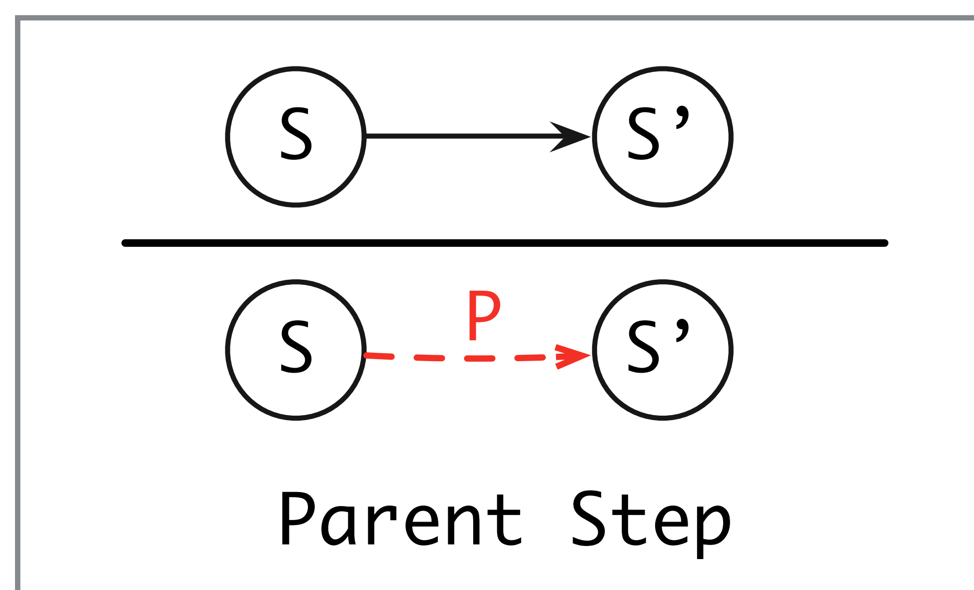


Lexical Scoping









Well formed path: R.P*.D

Shadowing

$\texttt{def } x_3 = z_2\ 5\ 7$

$\texttt{def } z_1 =$

$\quad \texttt{fun } x_1\ \{$

$\quad\quad \texttt{fun } y_1\ \{$

$\quad\quad\quad x_2 + y_2$

$\quad\quad \}$

$\quad \}$

Shadowing

S0

$\texttt{def}\ x_3 = z_2\ 5\ 7$

$\texttt{def}\ z_1 =$

S1

$\quad \texttt{fun}\ x_1\ \{$

S2

$\qquad \texttt{fun}\ y_1\ \{$

$\qquad\quad x_2 + y_2$

$\qquad \}$

$\quad \}$

Shadowing

def x_3 = z_2 5 7 S0

def z_1 =

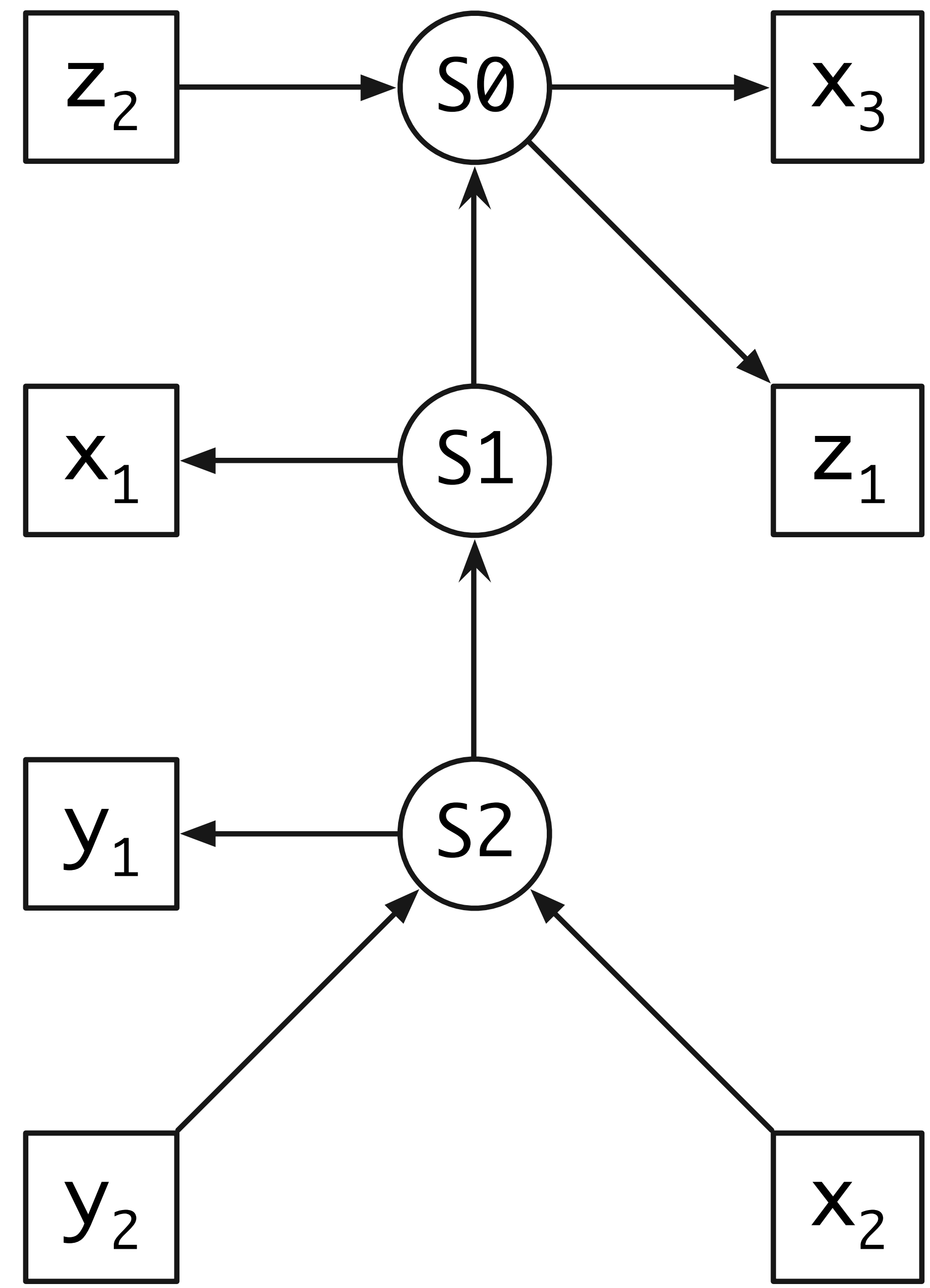
fun x_1 { S1

fun y_1 { S2

x_2 + y_2

}

}



Shadowing

S0

$\texttt{def}\ x_3 = z_2\ 5\ 7$

$\texttt{def}\ z_1 =$

S1

$\quad \texttt{fun}\ x_1\ \{$

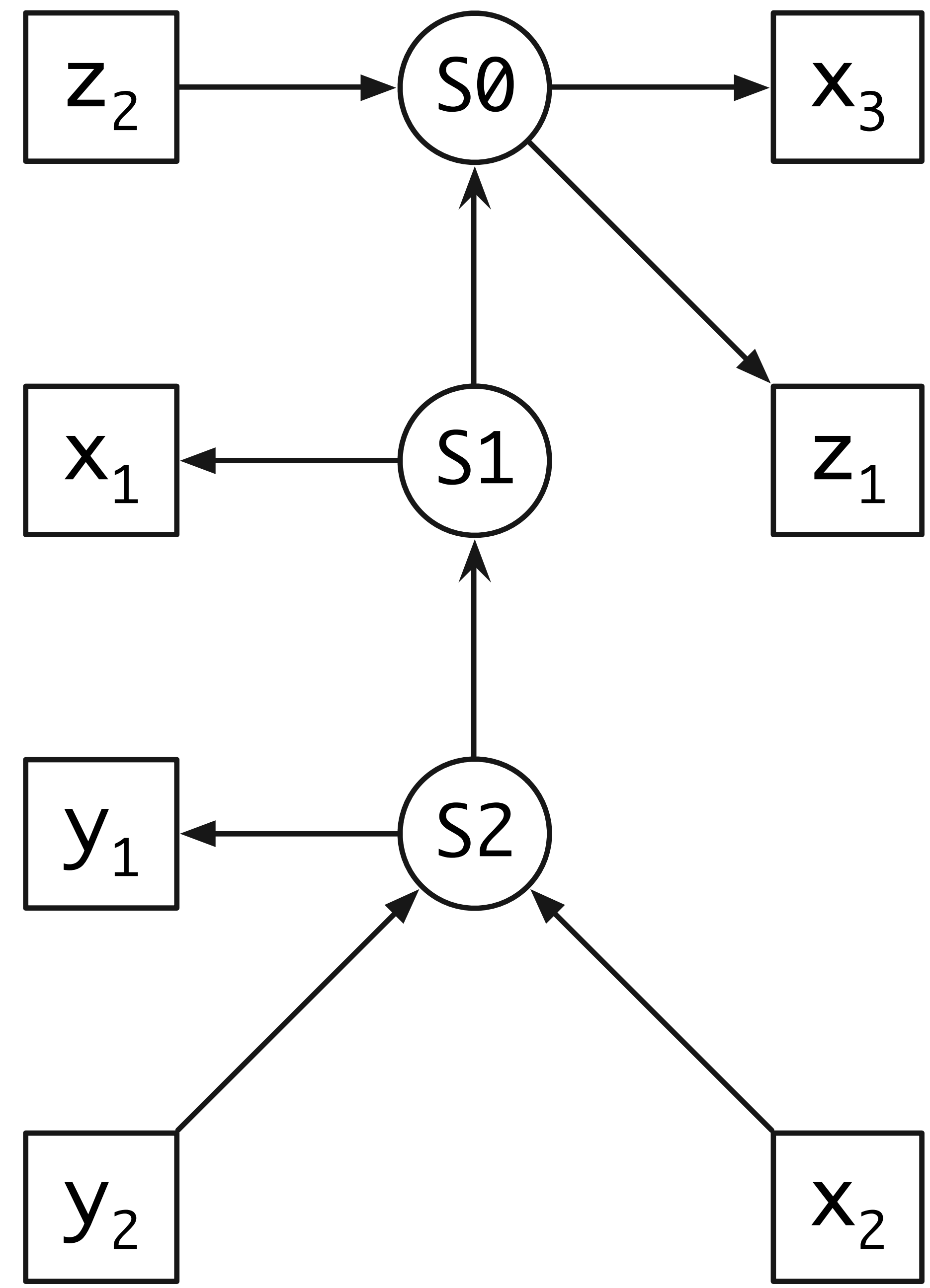
S2

$\quad\quad \texttt{fun}\ y_1\ \{$

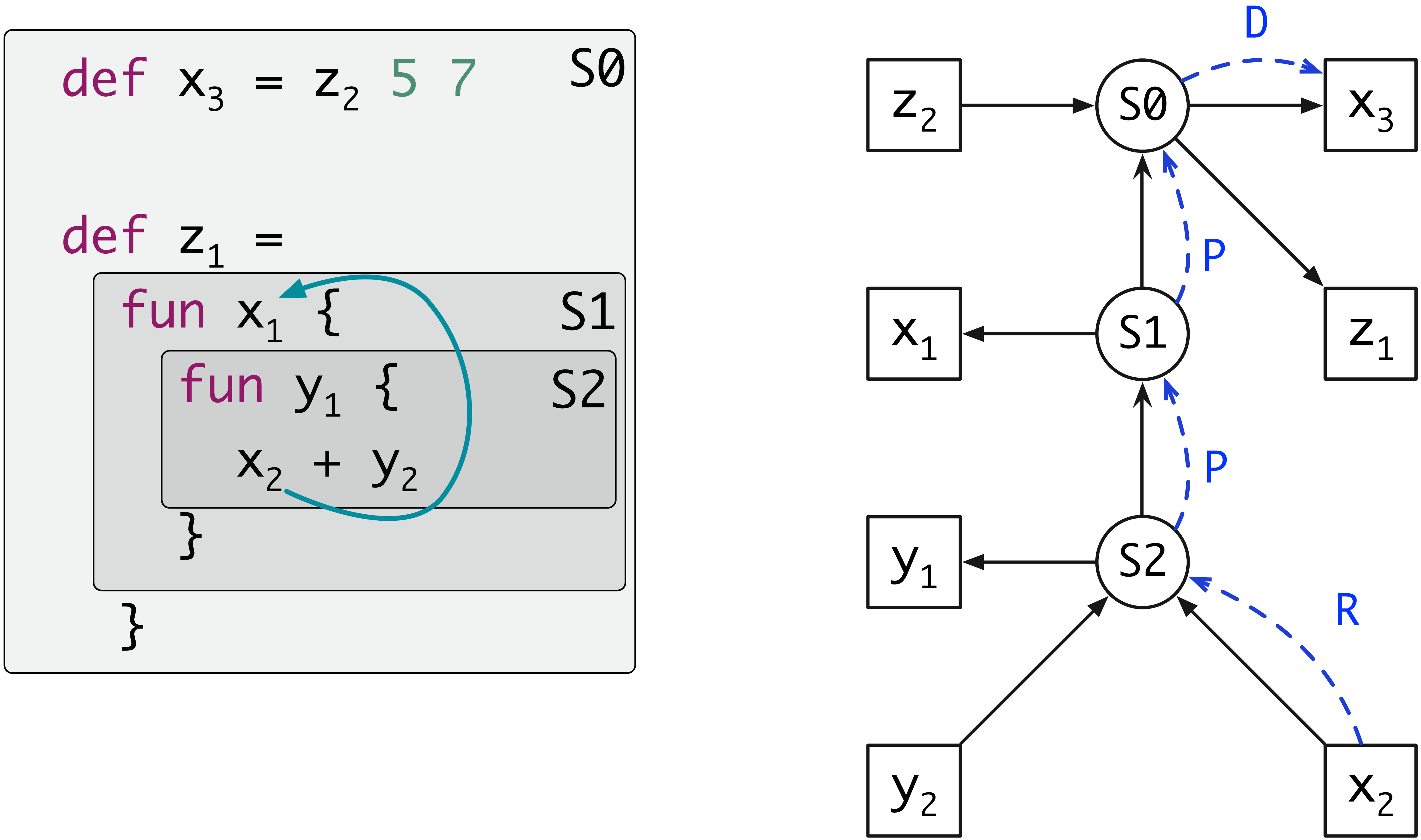
$\quad\quad\quad x_2 + y_2$

$\quad\quad \}$

$\quad \}$

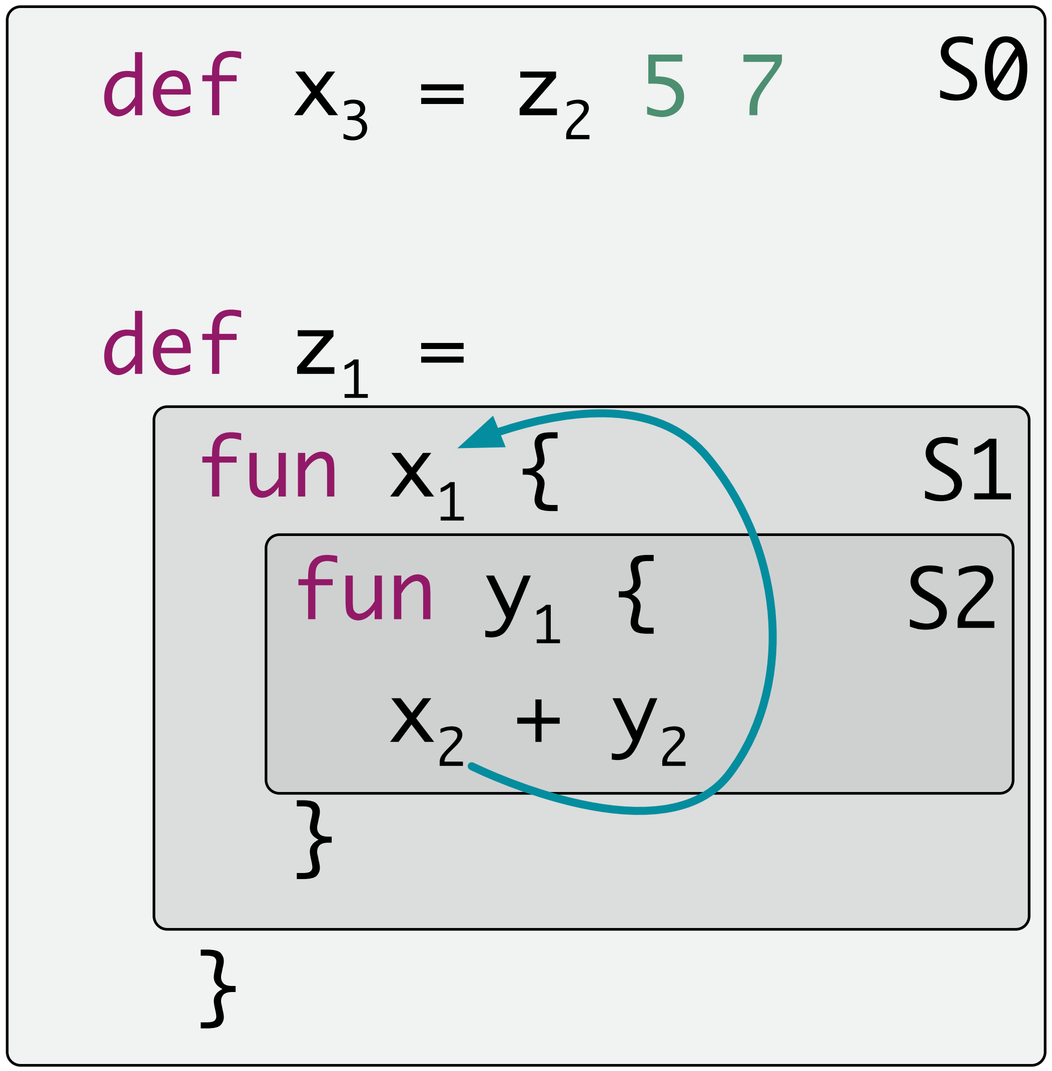


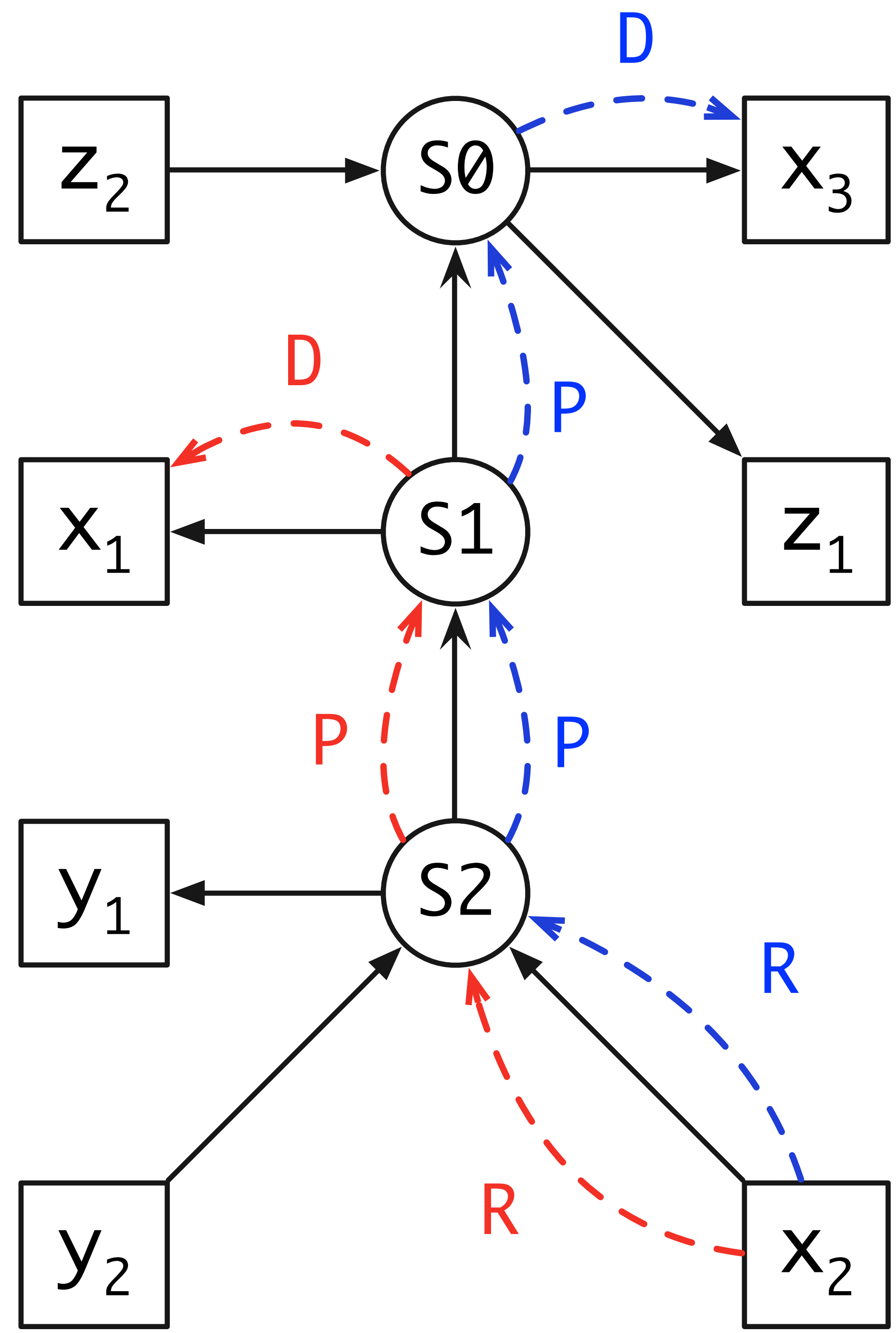
Shadowing



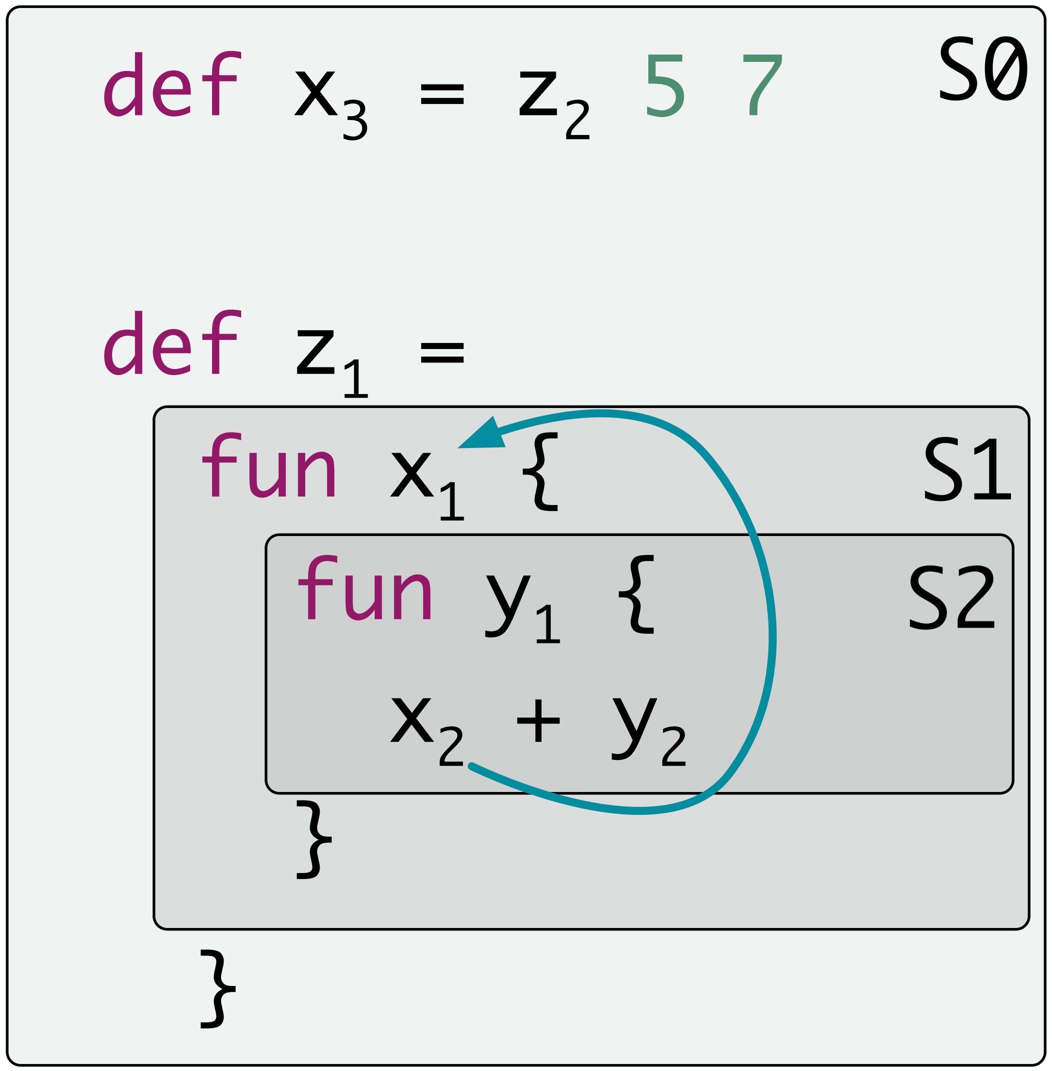
def x3 = z2 5 7
S0
def z1 =
fun x1 {
S1
fun y1 {
S2
x2 + y2
}
}
D
P
P
R
z2
S0
x3
x1
S1
z1
y1
S2
y2
x2

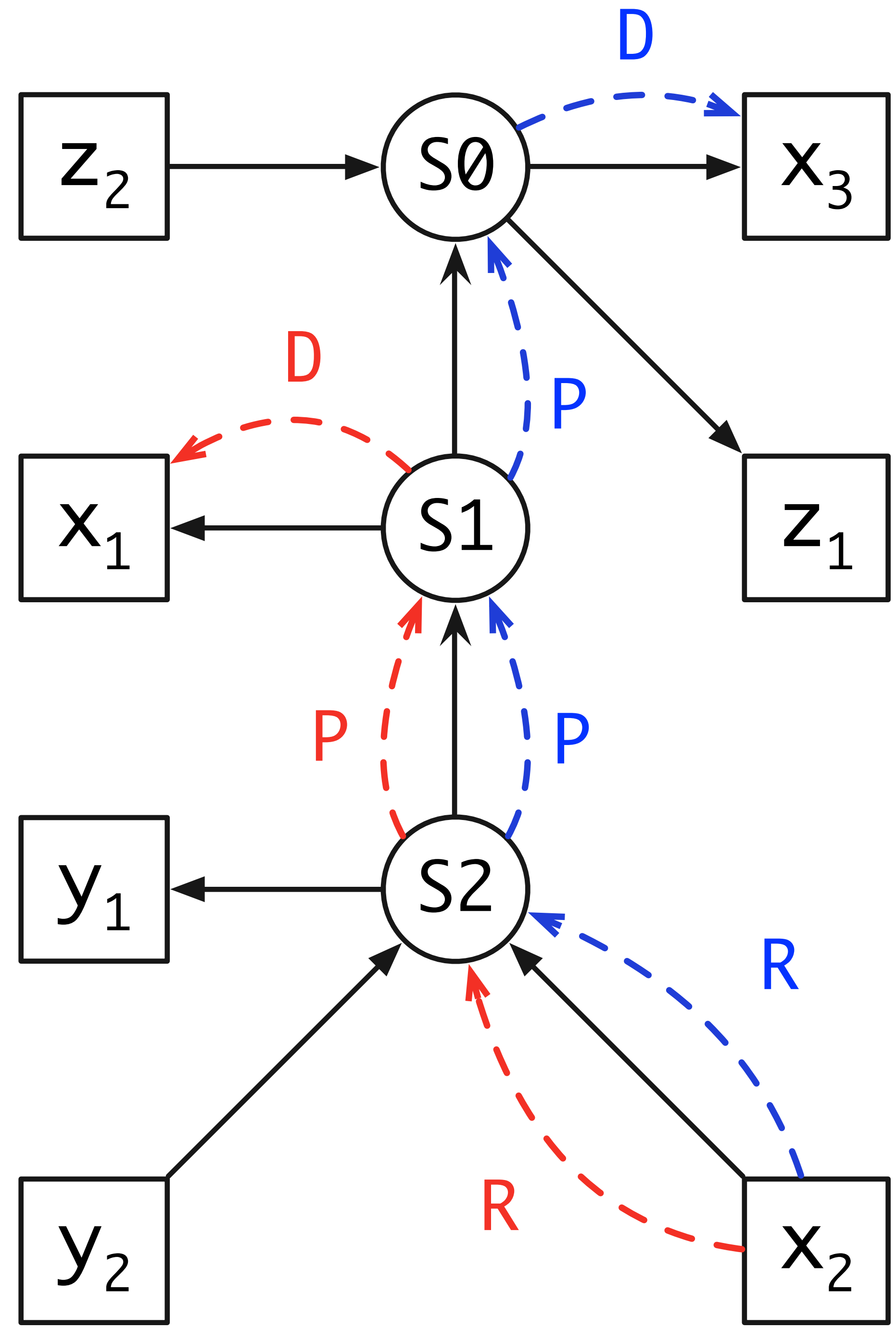
Shadowing


def x_3 = z_2 5 7
S0
def z_1 =
fun x_1 {
S1
fun y_1 {
S2
x_2 + y_2
}
}


z_2
S0
x_3
D
D
P
x_1
S1
z_1
P
P
y_1
S2
R
R
y_2
x_2

Shadowing



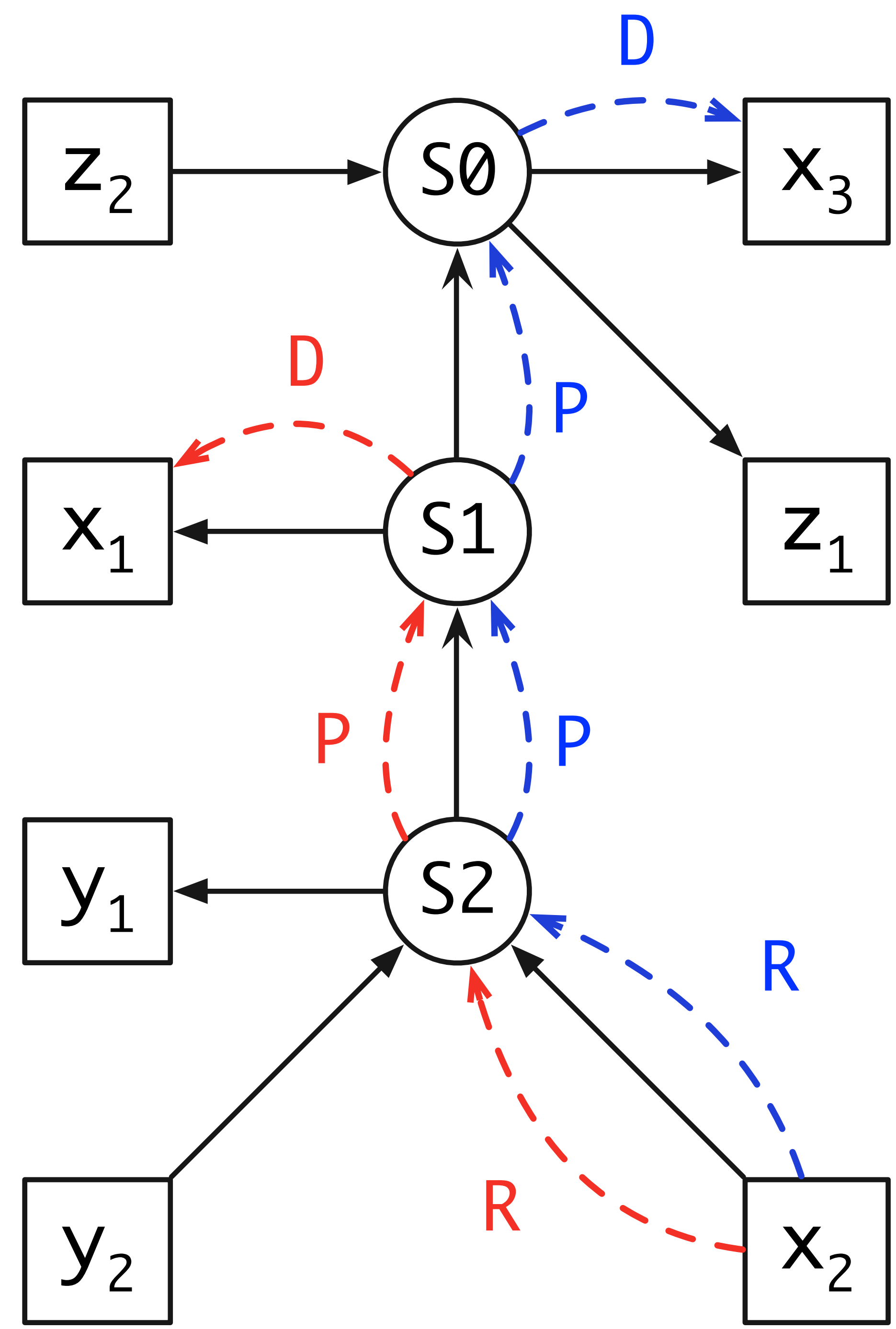


D < P.p

Shadowing

```
def x_3 = z_2 5 7                S0

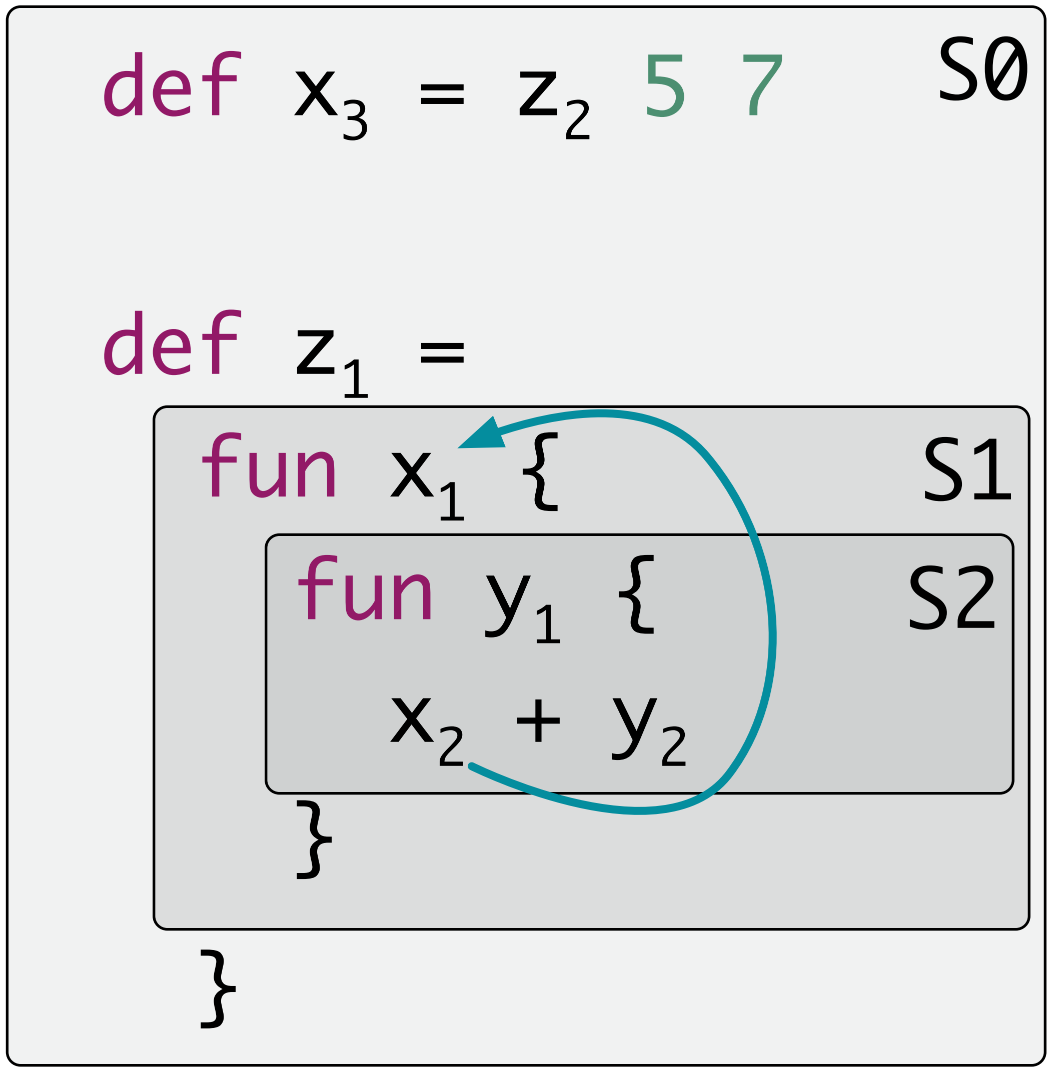
def z_1 =
  fun x_1 {                      S1
    fun y_1 {                    S2
      x_2 + y_2
    }
  }
```



$$\frac{}{D < P.p}$$

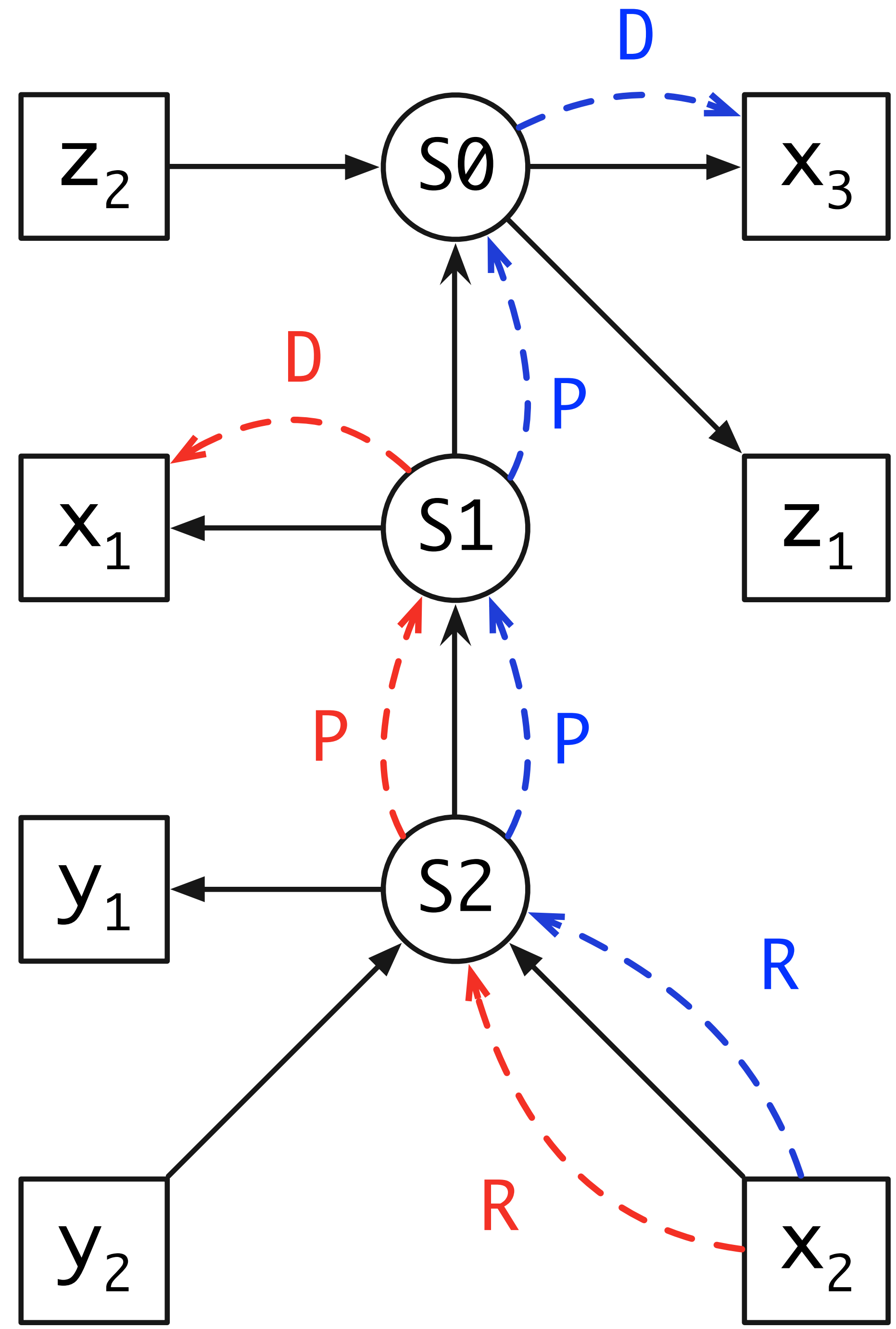
$$\frac{p < p'}{s.p < s.p'}$$

Shadowing



$$\frac{}{D < P.p}$$

$$\frac{p < p'}{s.p < s.p'}$$



$$R.P.D < R.P.P.D$$

Imports

S0

module A_1 {

  def z_1 = 5   SA

}

module B_1 {

  import A_2 SB

  def x_1 = 1 + z_2

}

Imports

S0

module A_1 {

  def z_1 = 5   SA

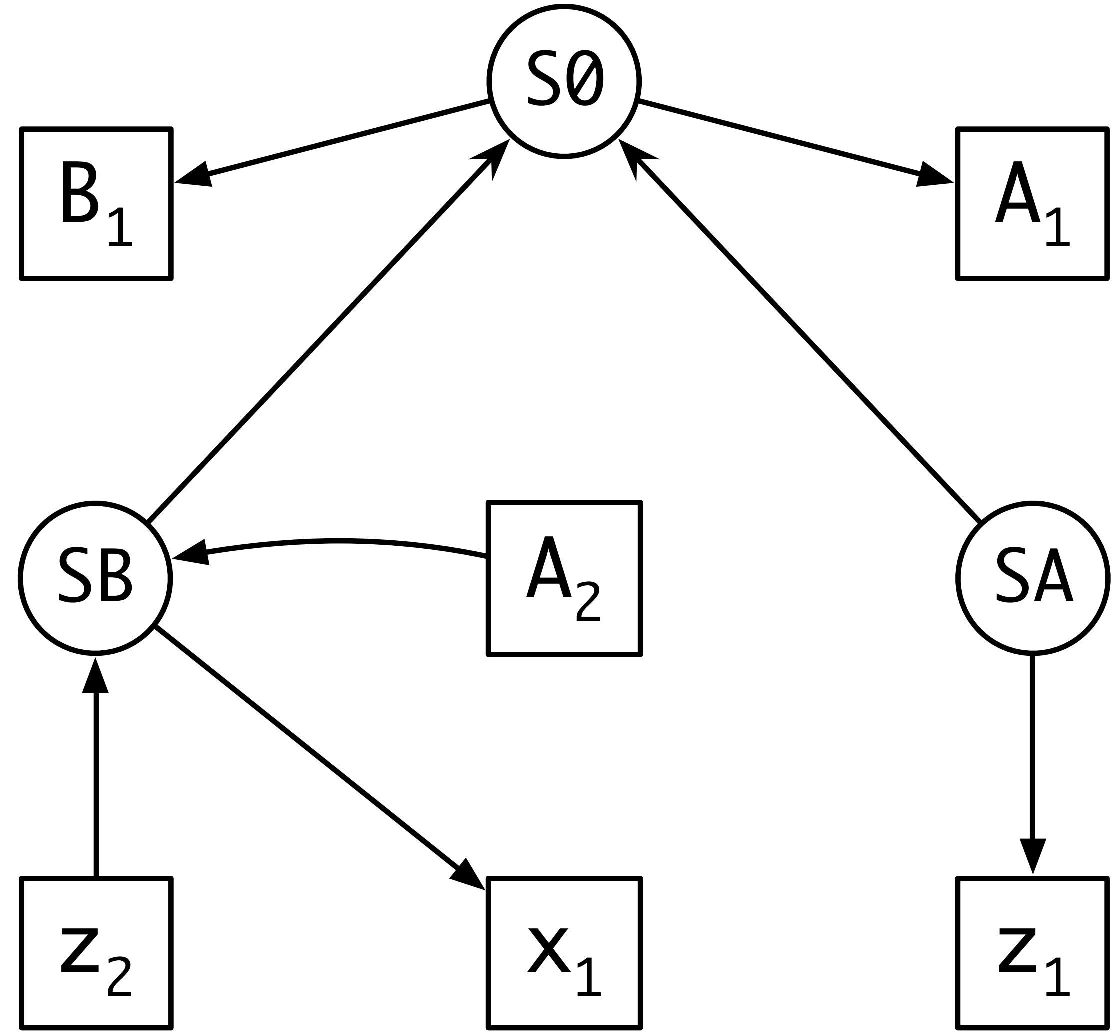
}

module B_1 {

  import A_2 SB

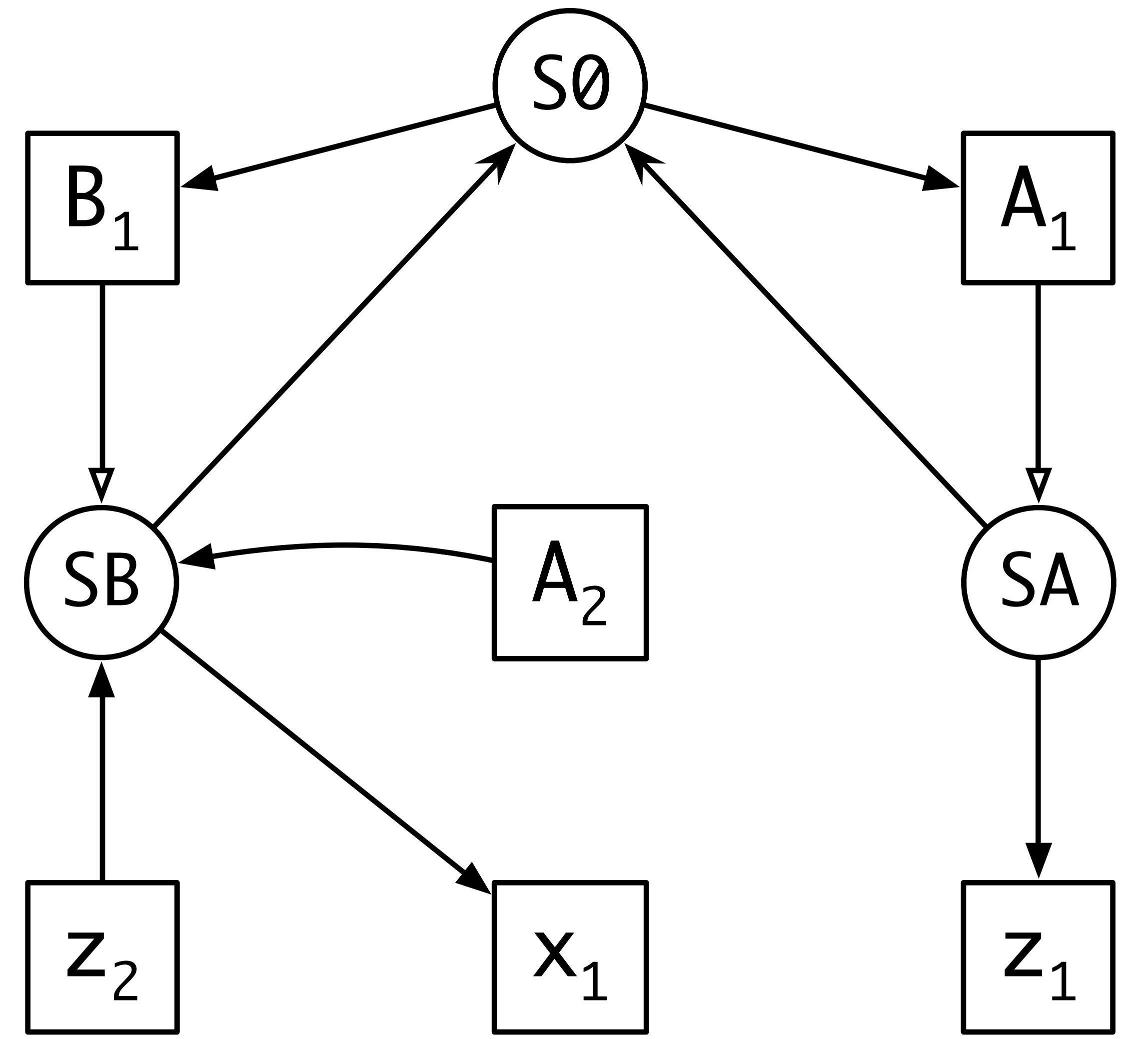
  def x_1 = 1 + z_2

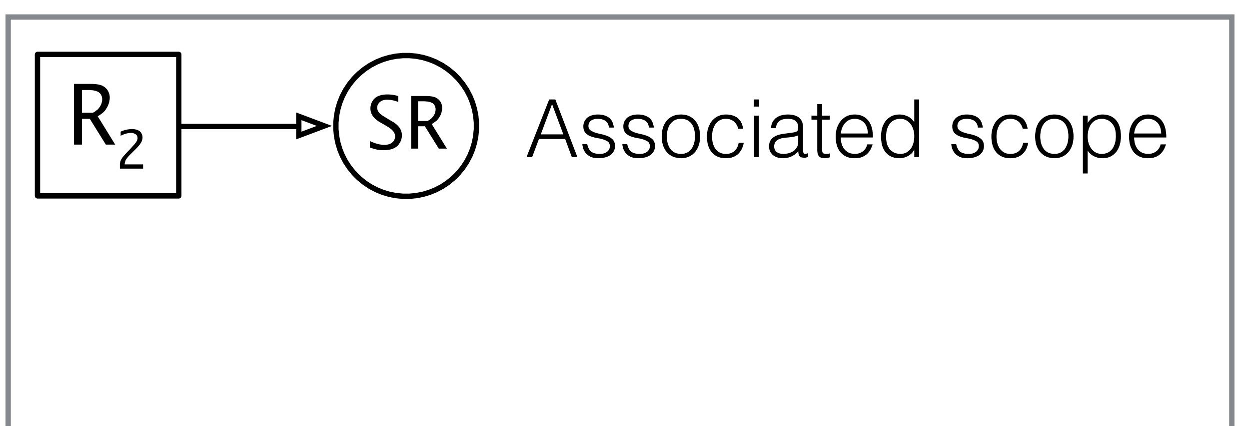
}



Imports

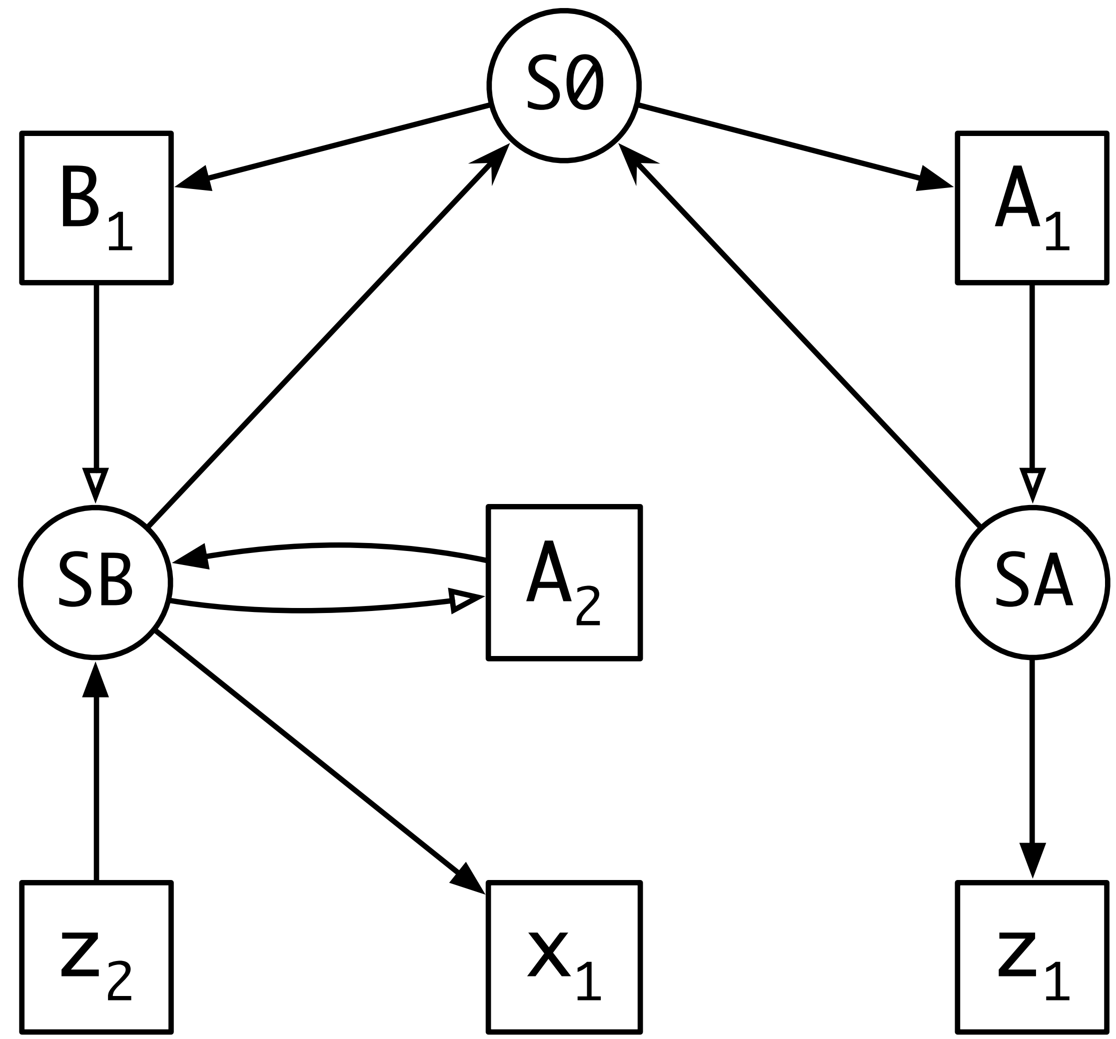
```
module A1 {                S0
  def z1 = 5               SA
}
module B1 {
  import A2                SB
  def x1 = 1 + z2
}
```

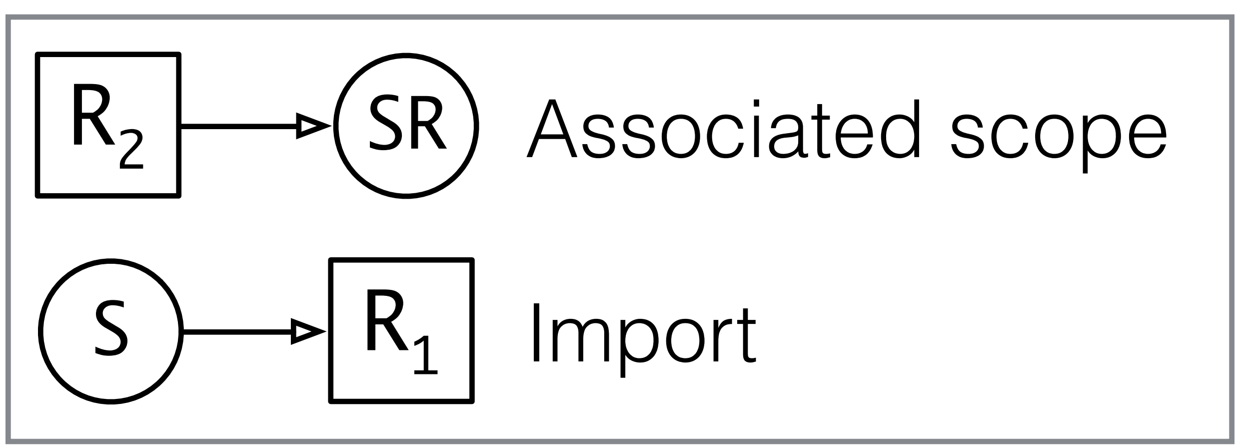




Imports

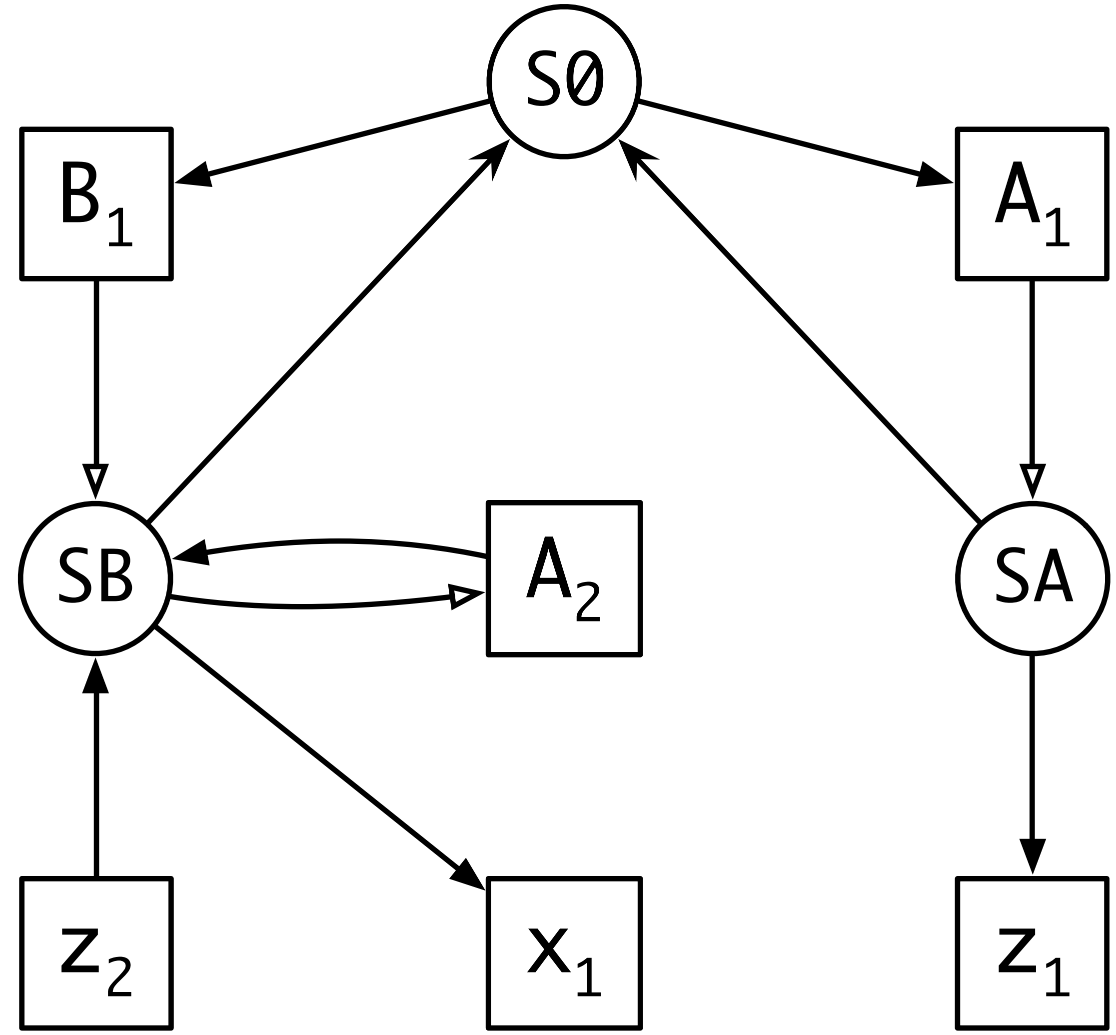
```
module A₁ {                S0
  def z₁ = 5           SA
}
module B₁ {
  import A₂            SB
  def x₁ = 1 + z₂
}
```

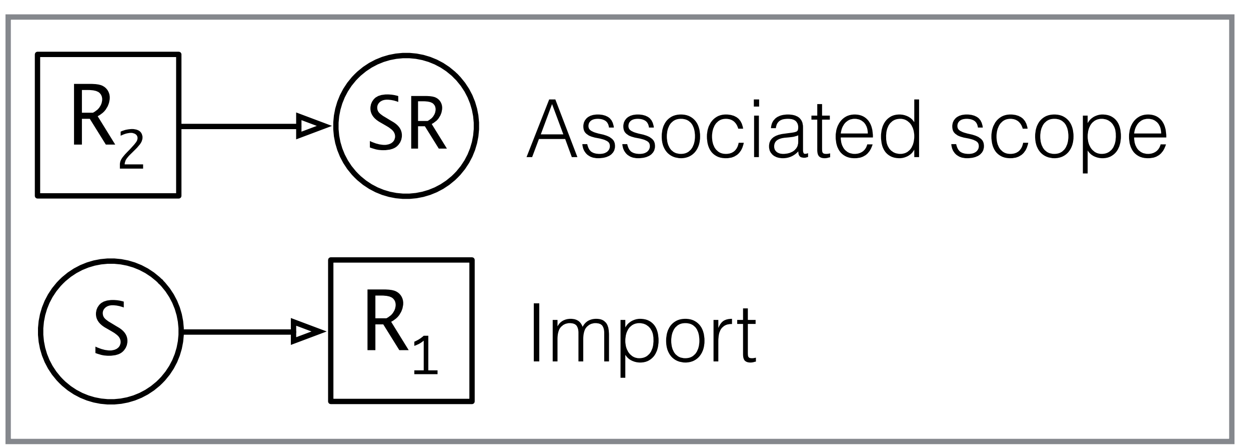




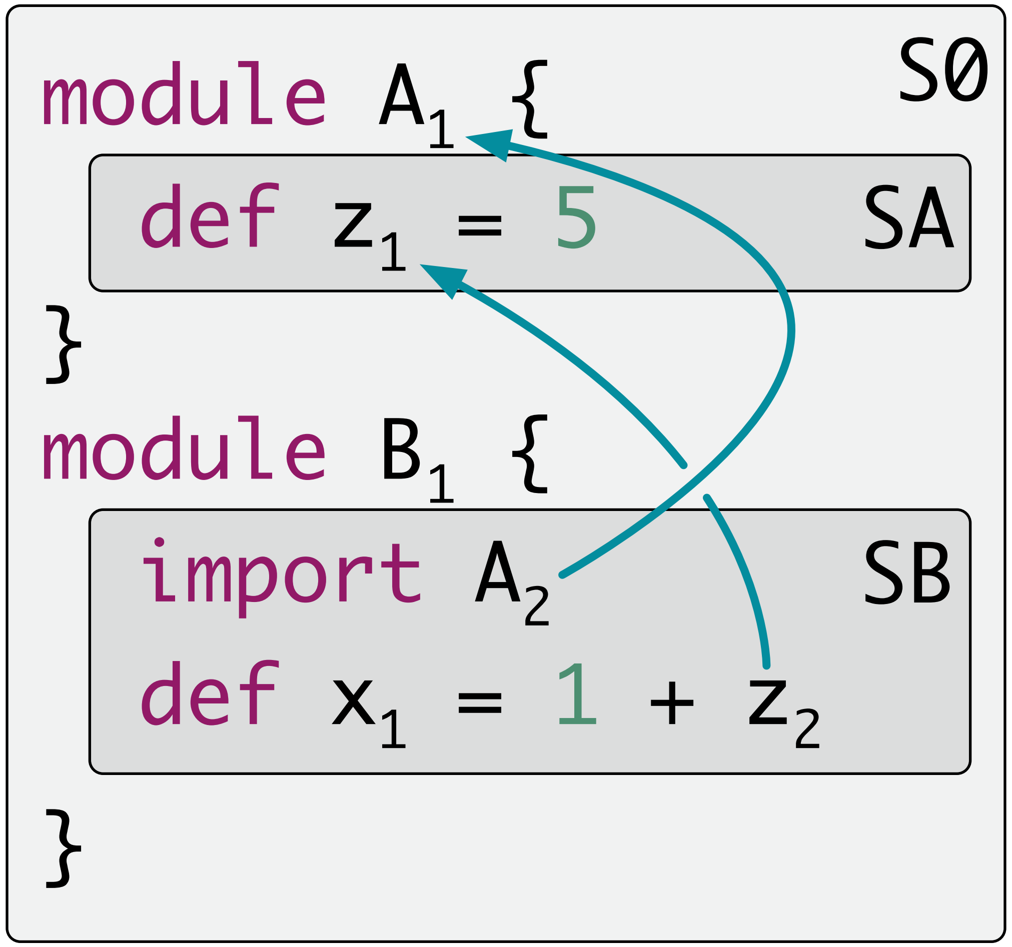
Imports

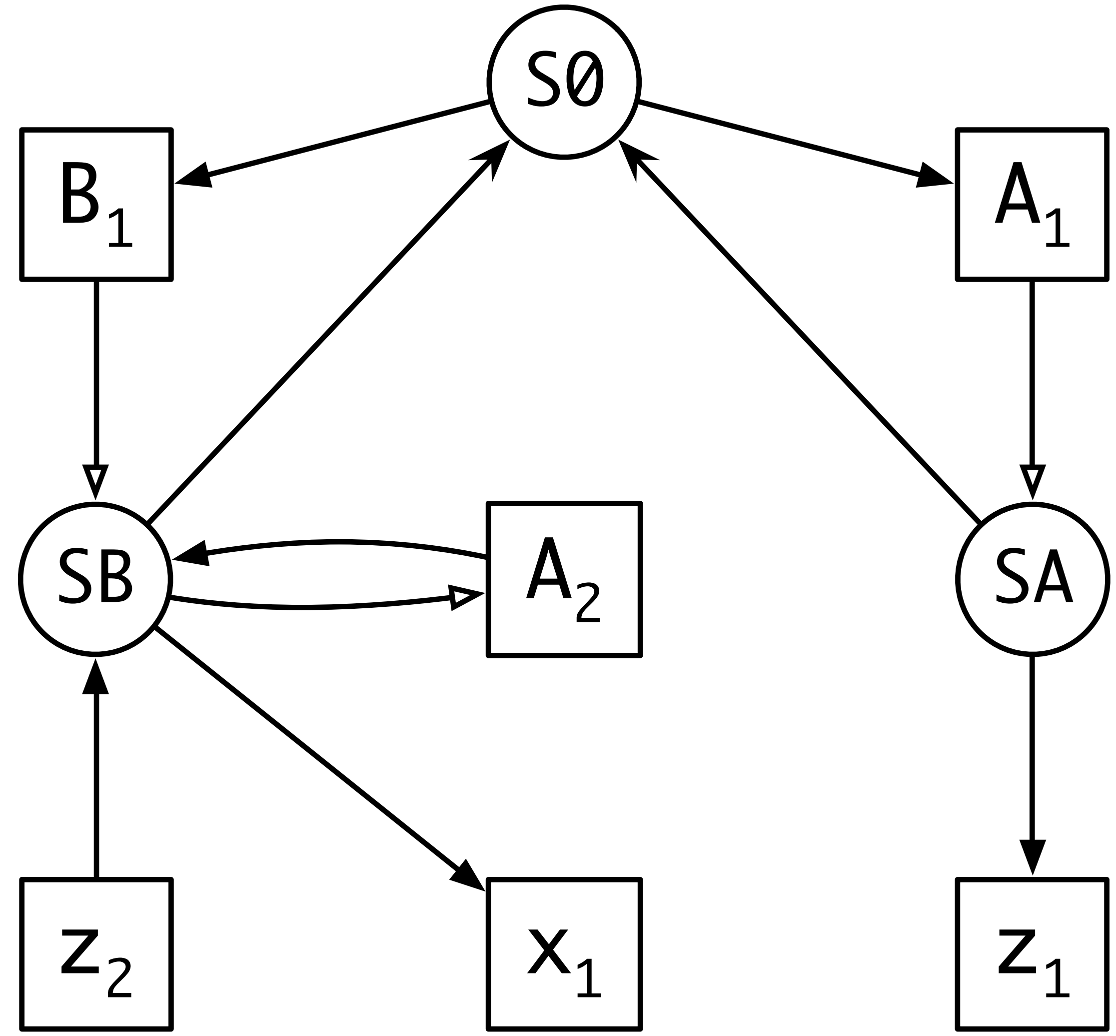
```
module A₁ {                S0
  def z₁ = 5               SA
}
module B₁ {
  import A₂                SB
  def x₁ = 1 + z₂
}
```

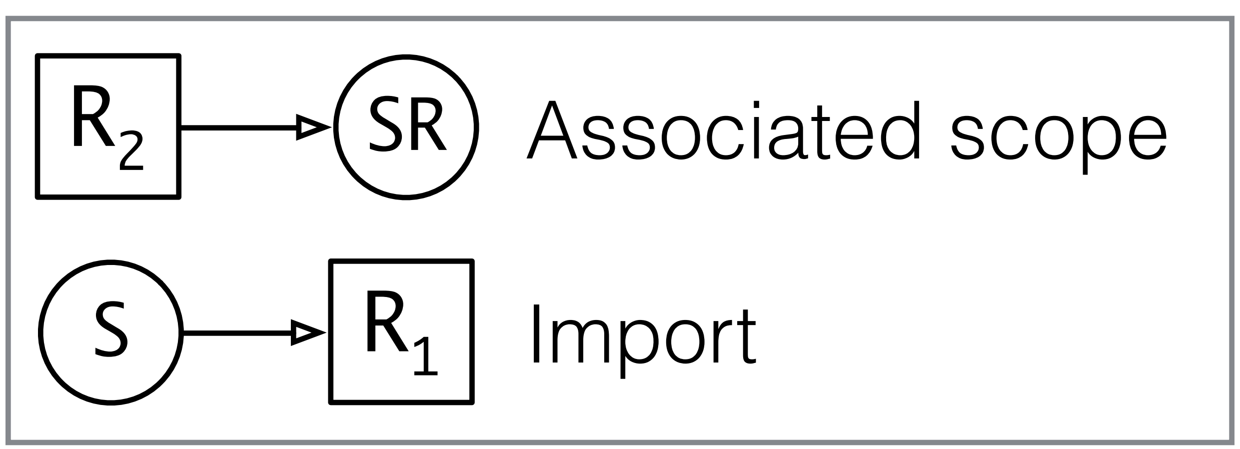




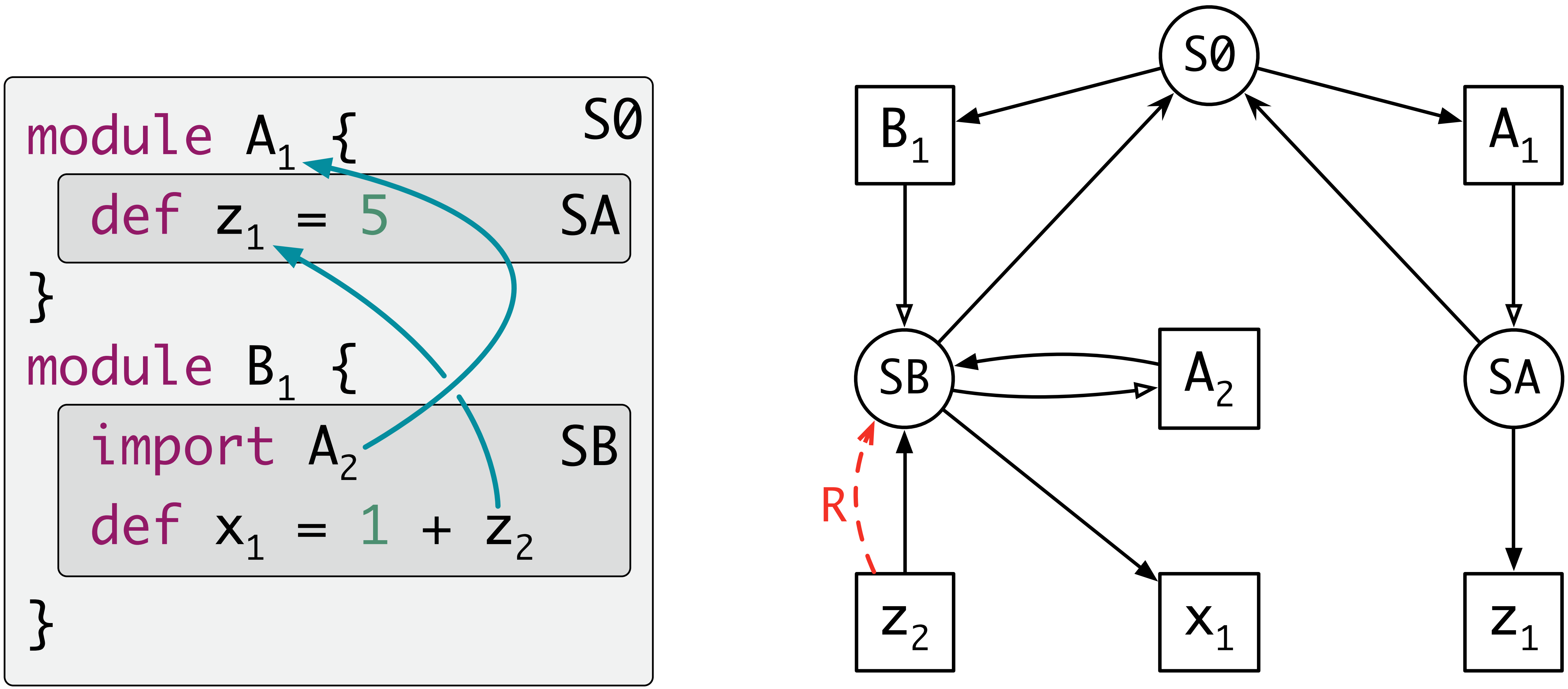
Imports

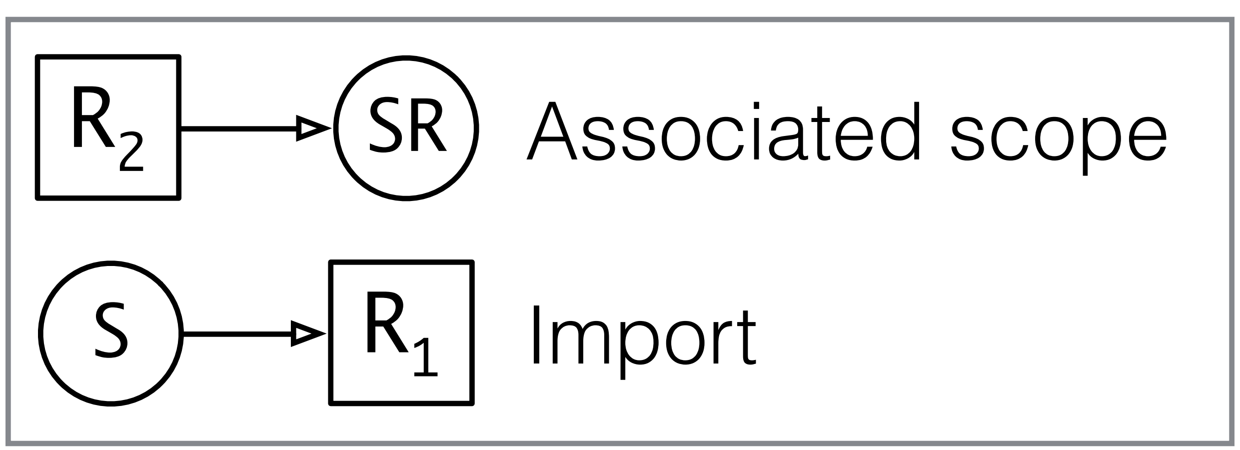

module A1 {
def z1 = 5
}
module B1 {
import A2
def x1 = 1 + z2
}
S0
SA
SB


S0
B1
A1
SB
A2
SA
z2
x1
z1

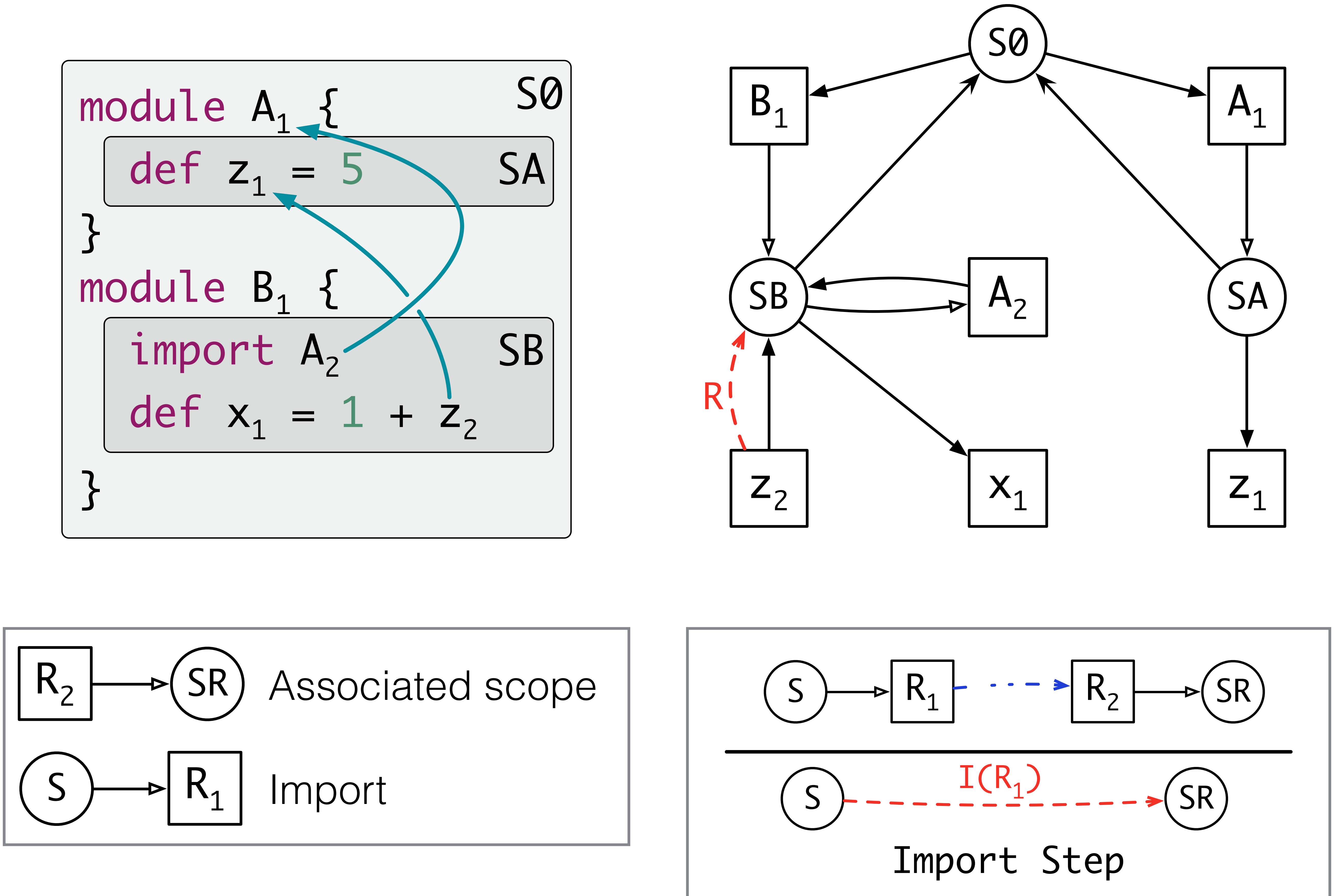

R2
SR
Associated scope
S
R1
Import

Imports


module A1 {
def z1 = 5
SA
}
module B1 {
import A2
def x1 = 1 + z2
SB
}
S0
S0
B1
A1
SB
A2
SA
R
z2
x1
z1

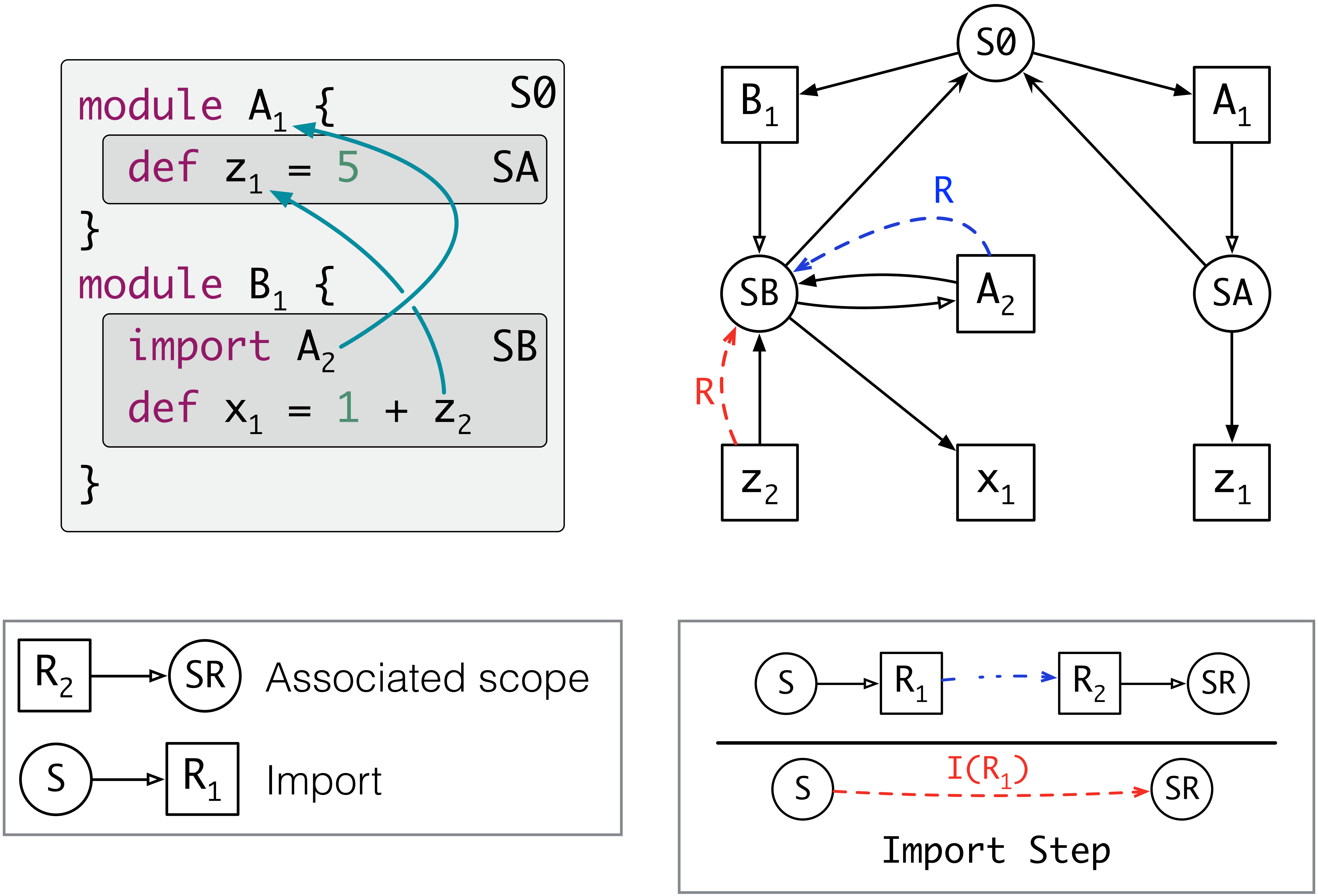

R2
SR
Associated scope
S
R1
Import

Imports



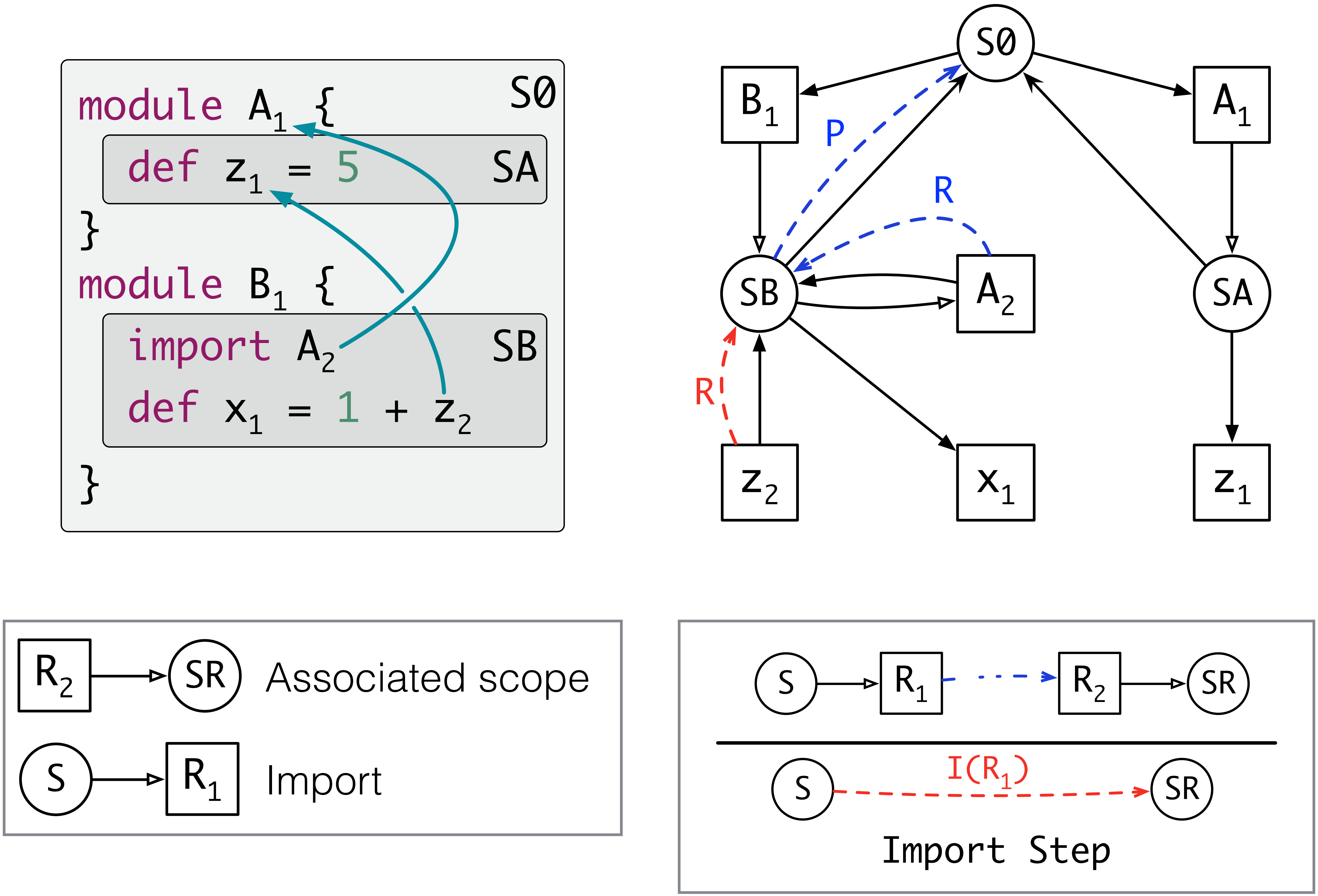
module A1 {
def z1 = 5
SA
}
S0
module B1 {
import A2
def x1 = 1 + z2
SB
}
S0
B1
A1
SB
A2
SA
R
z2
x1
z1
R2
SR
Associated scope
S
R1
Import
S
R1
R2
SR
I(R1)
S
SR
Import Step

Imports

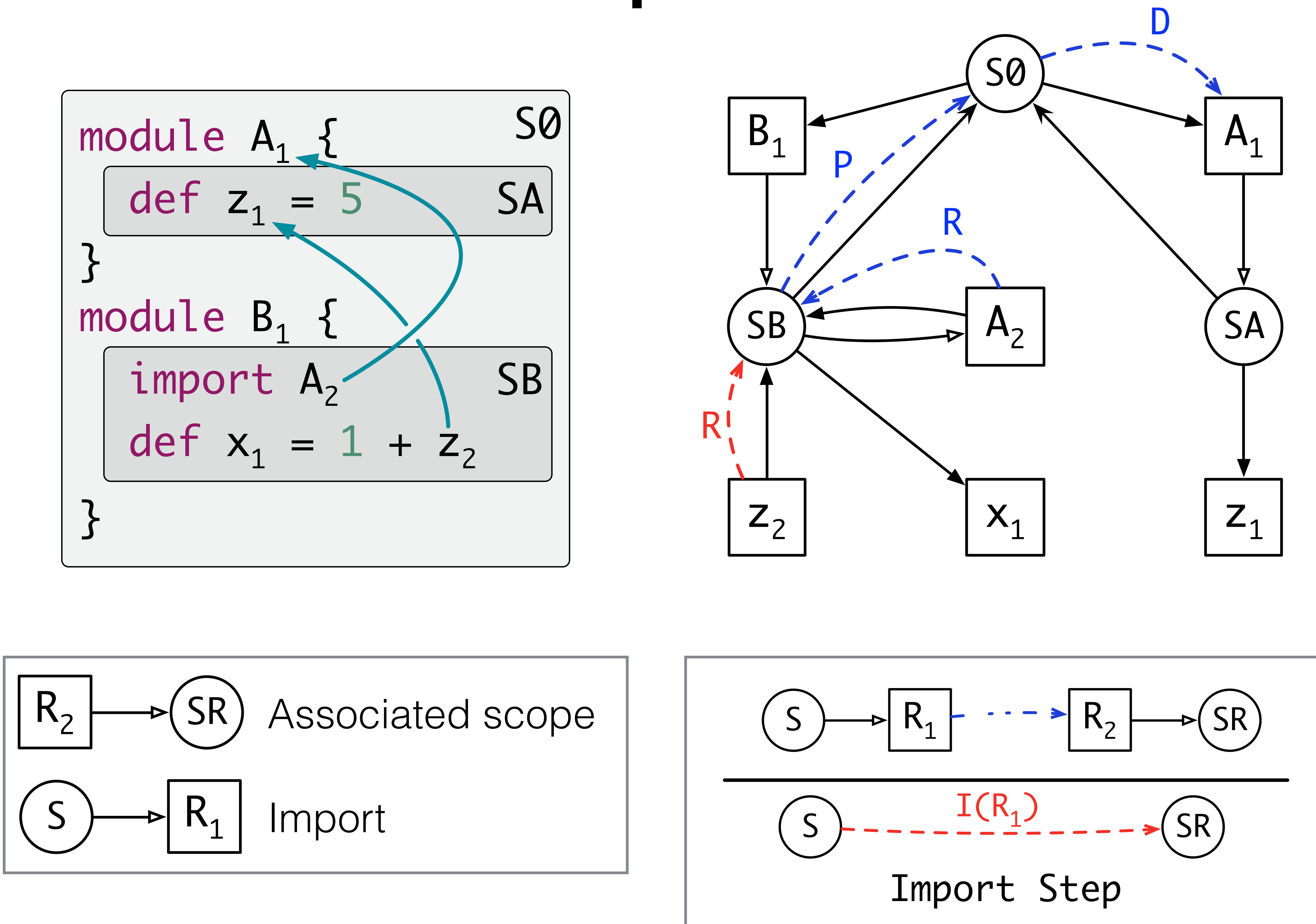


module A1 {
def z1 = 5
SA
}
S0
module B1 {
import A2
def x1 = 1 + z2
SB
}
S0
B1
A1
SB
A2
SA
R
R
z2
x1
z1
R2
SR
Associated scope
S
R1
Import
S
R1
R2
SR
S
I(R1)
SR
Import Step

Imports

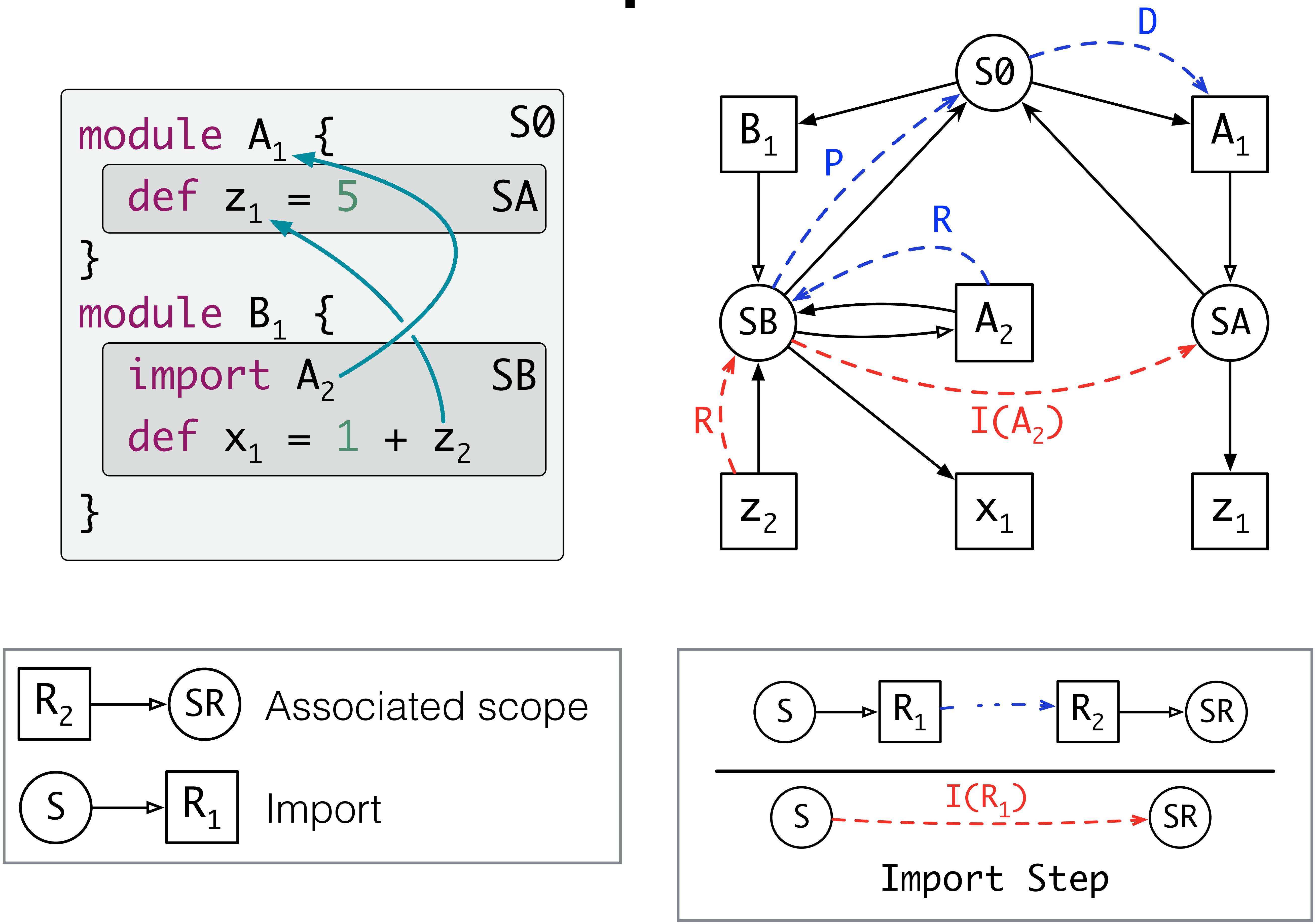

module A1 {
def z1 = 5
}
module B1 {
import A2
def x1 = 1 + z2
}
S0
SA
SB
B1
A1
A2
SB
SA
z2
x1
z1
P
R
R
Associated scope
Import
S
R1
R2
SR
I(R1)
Import Step

Imports



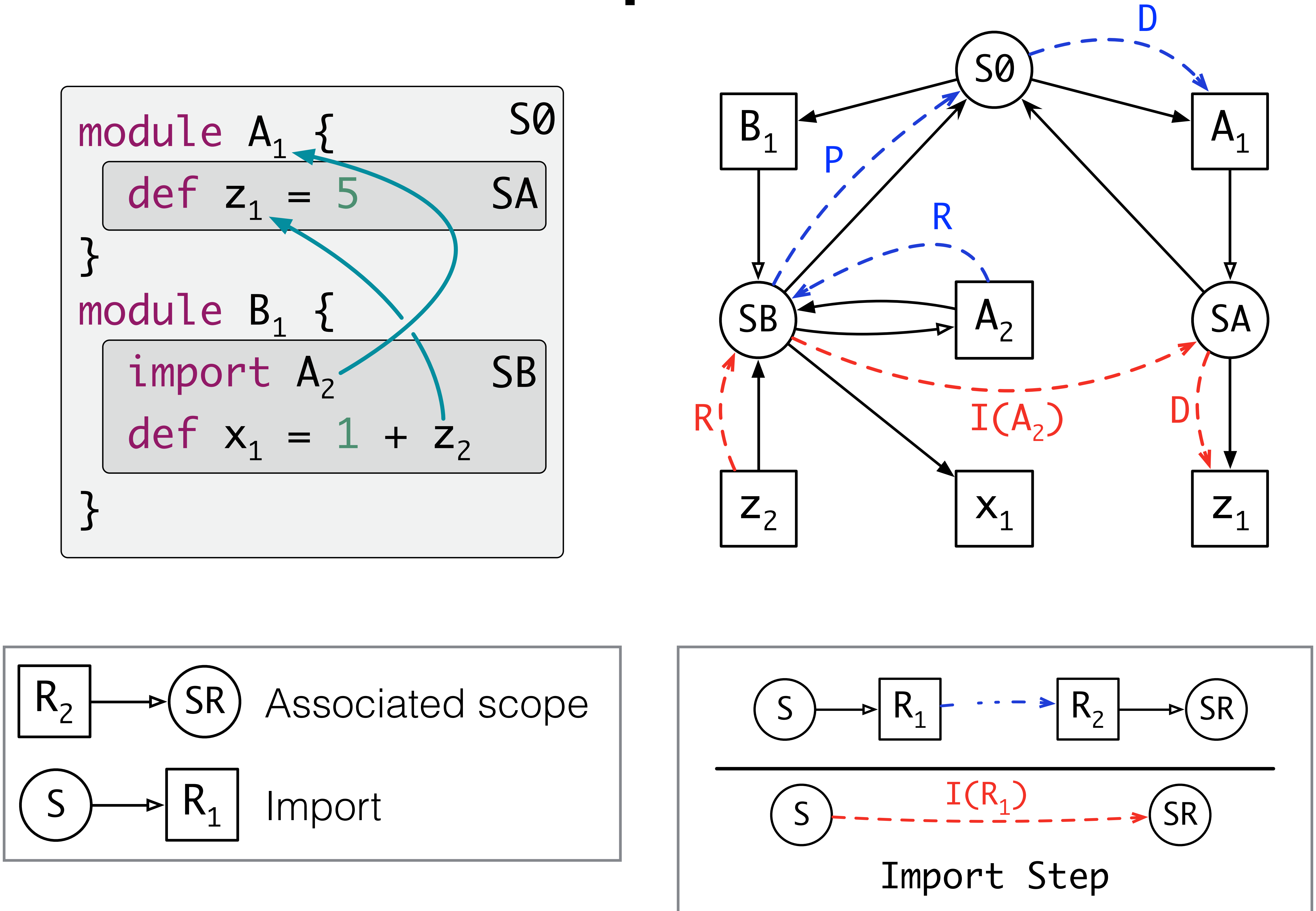
module A1 {
def z1 = 5
}
module B1 {
import A2
def x1 = 1 + z2
}
S0
SA
SB
D
P
R
R
S0
B1
A1
SB
A2
SA
z2
x1
z1
R2
SR
Associated scope
S
R1
Import
S
R1
R2
SR
S
I(R1)
SR
Import Step

Imports



module A1 {
def z1 = 5
}
module B1 {
import A2
def x1 = 1 + z2
}
S0
SA
SB
D
P
R
R
I(A2)
B1
A1
SB
A2
SA
z2
x1
z1
R2
SR
Associated scope
S
R1
Import
S
R1
R2
SR
S
I(R1)
SR
Import Step

Imports



module A1 {
def z1 = 5
SA
}
S0
module B1 {
import A2
def x1 = 1 + z2
SB
}
S0
B1
A1
SB
A2
SA
z2
x1
z1
D
P
R
R
I(A2)
D
R2
SR
Associated scope
S
R1
Import
S
R1
R2
SR
S
SR
I(R1)
Import Step

Imports shadow Parents

S0

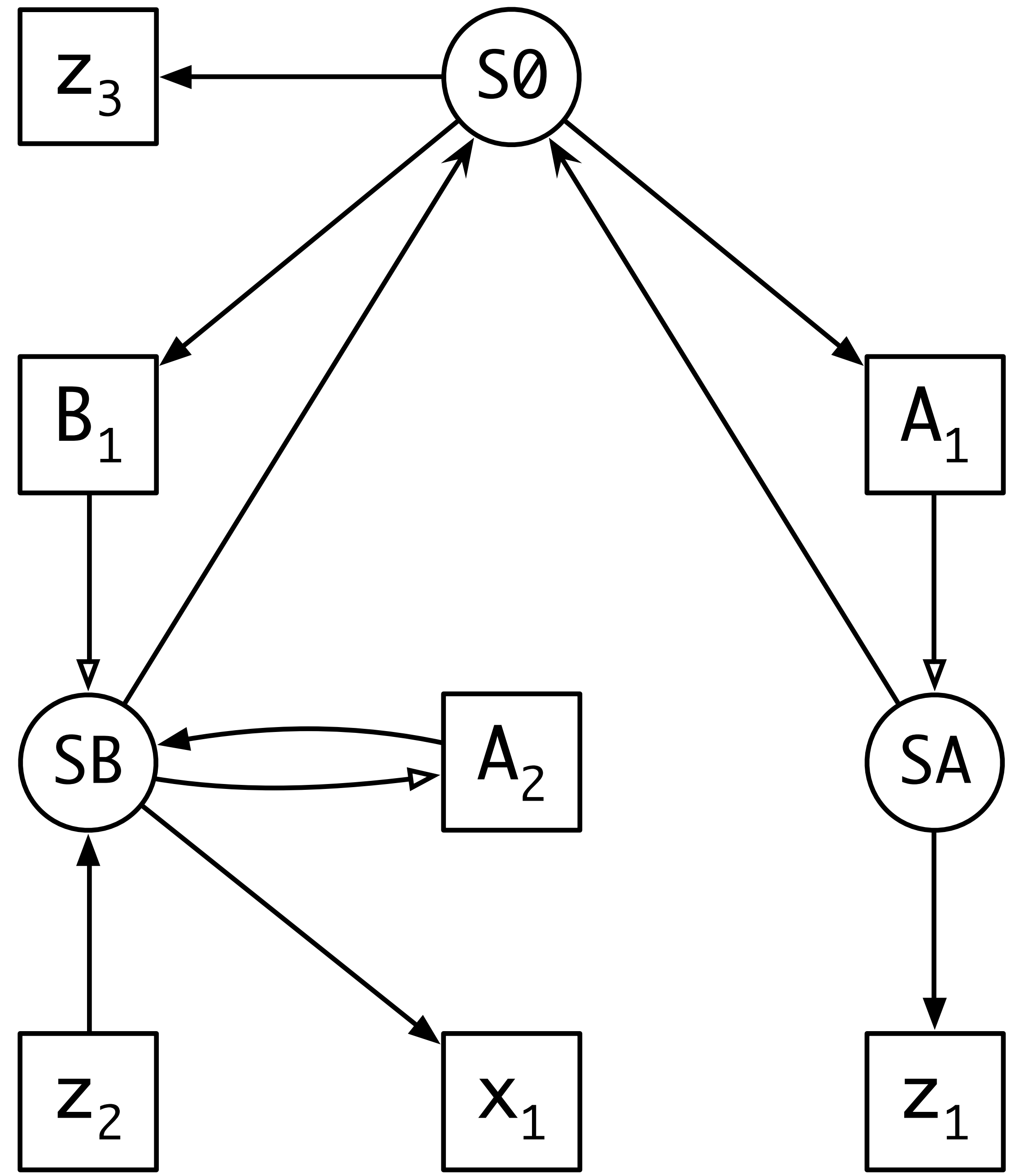
```
def z3 = 2

module A1 {
  def z1 = 5          SA
}
module B1 {
  import A2           SB
  def x1 = 1 + z2
}
```

Imports shadow Parents

```
def z3 = 2                    S0

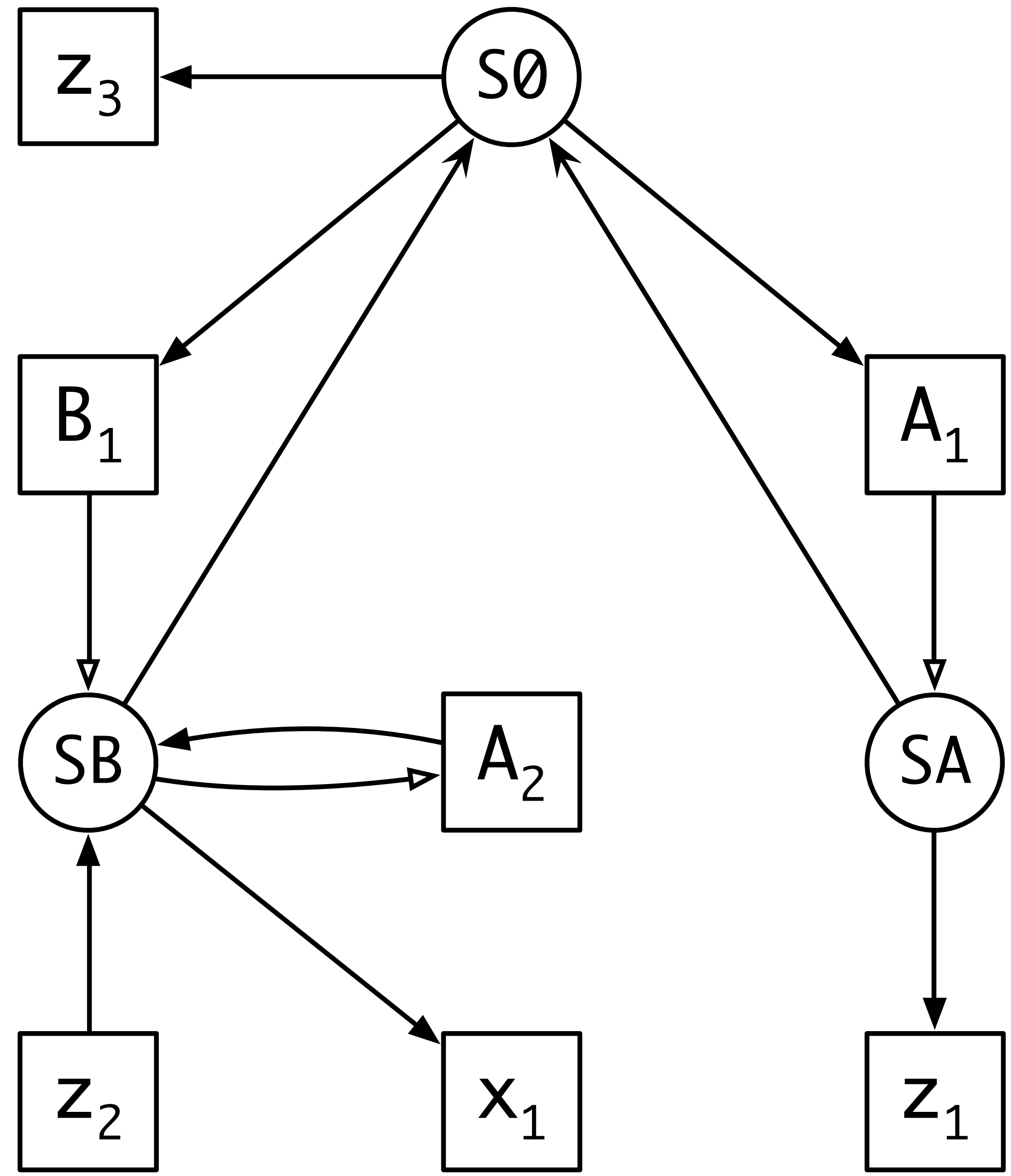
module A1 {
    def z1 = 5                SA
}
module B1 {
    import A2                 SB
    def x1 = 1 + z2
}
```



Imports shadow Parents

```
def z_3 = 2                              S0

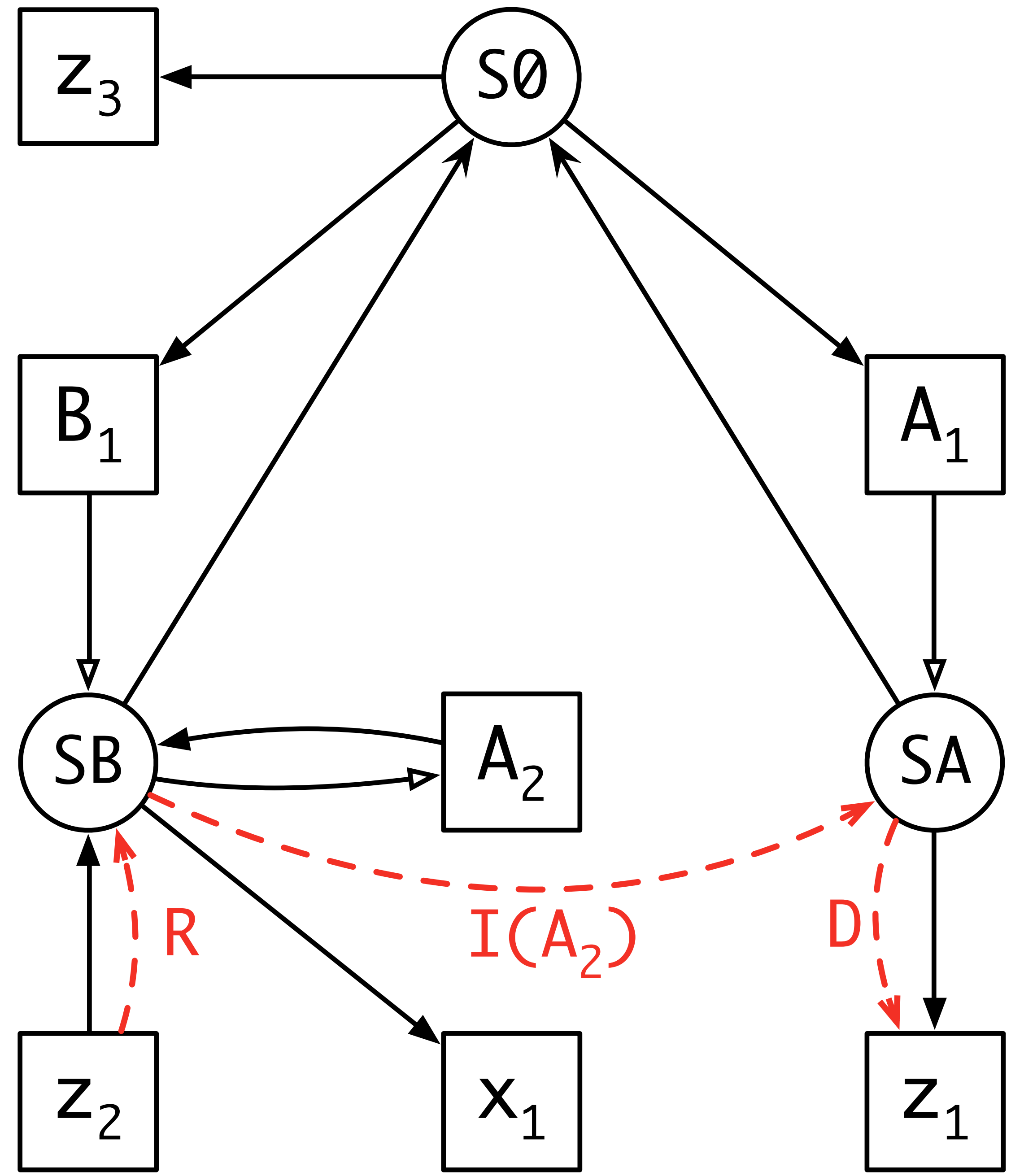
module A_1 {
    def z_1 = 5                          SA
}
module B_1 {
    import A_2                           SB
    def x_1 = 1 + z_2
}
```



Imports shadow Parents

```
def z3 = 2                                  S0

module A1 {
    def z1 = 5                              SA
}
module B1 {
    import A2                               SB
    def x1 = 1 + z2
}
```



Imports shadow Parents

S0

def $z_3 = 2$

module A_1 {

SA

def $z_1 = 5$

}

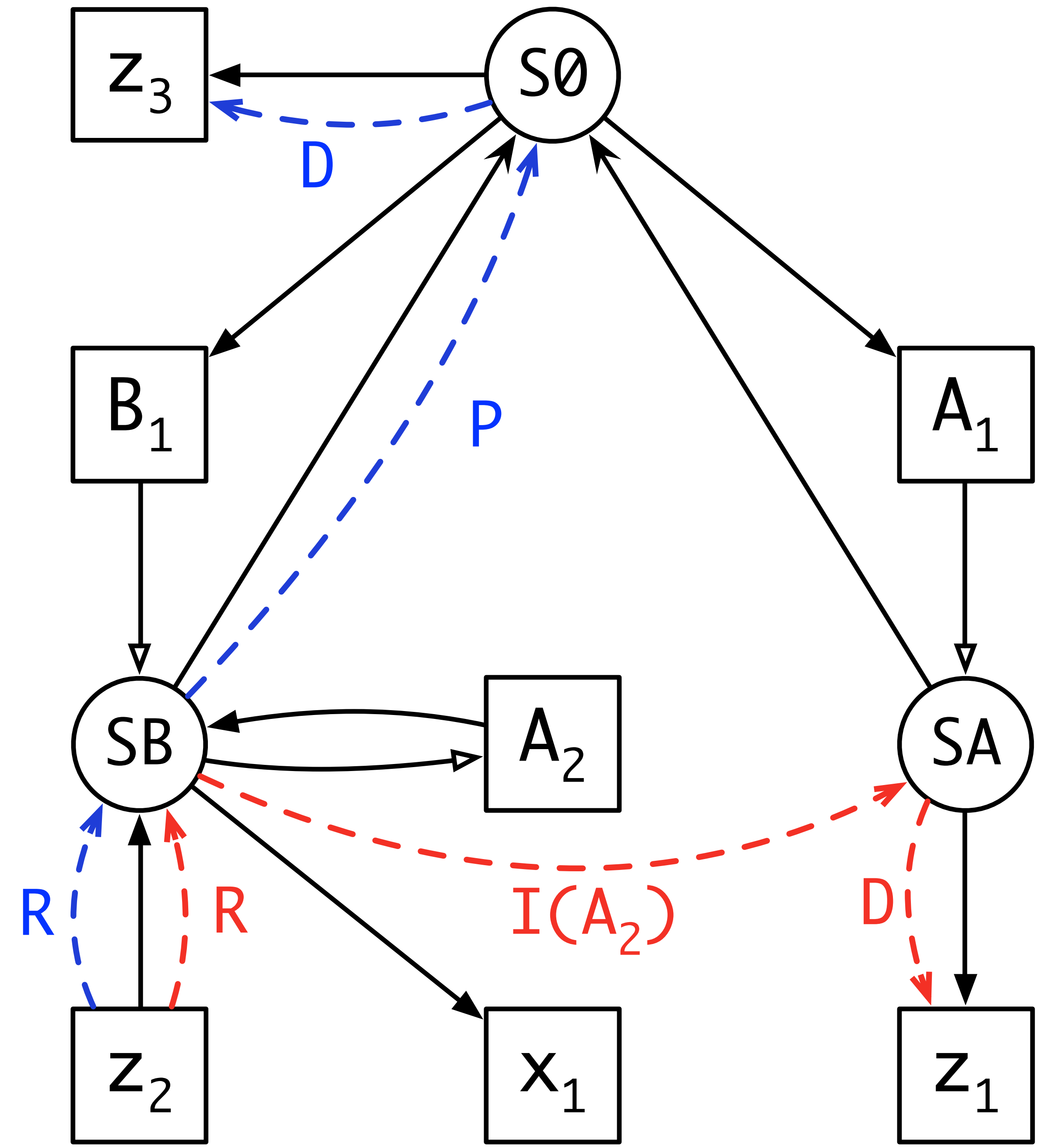
module B_1 {

SB

import A_2

def $x_1 = 1 + z_2$

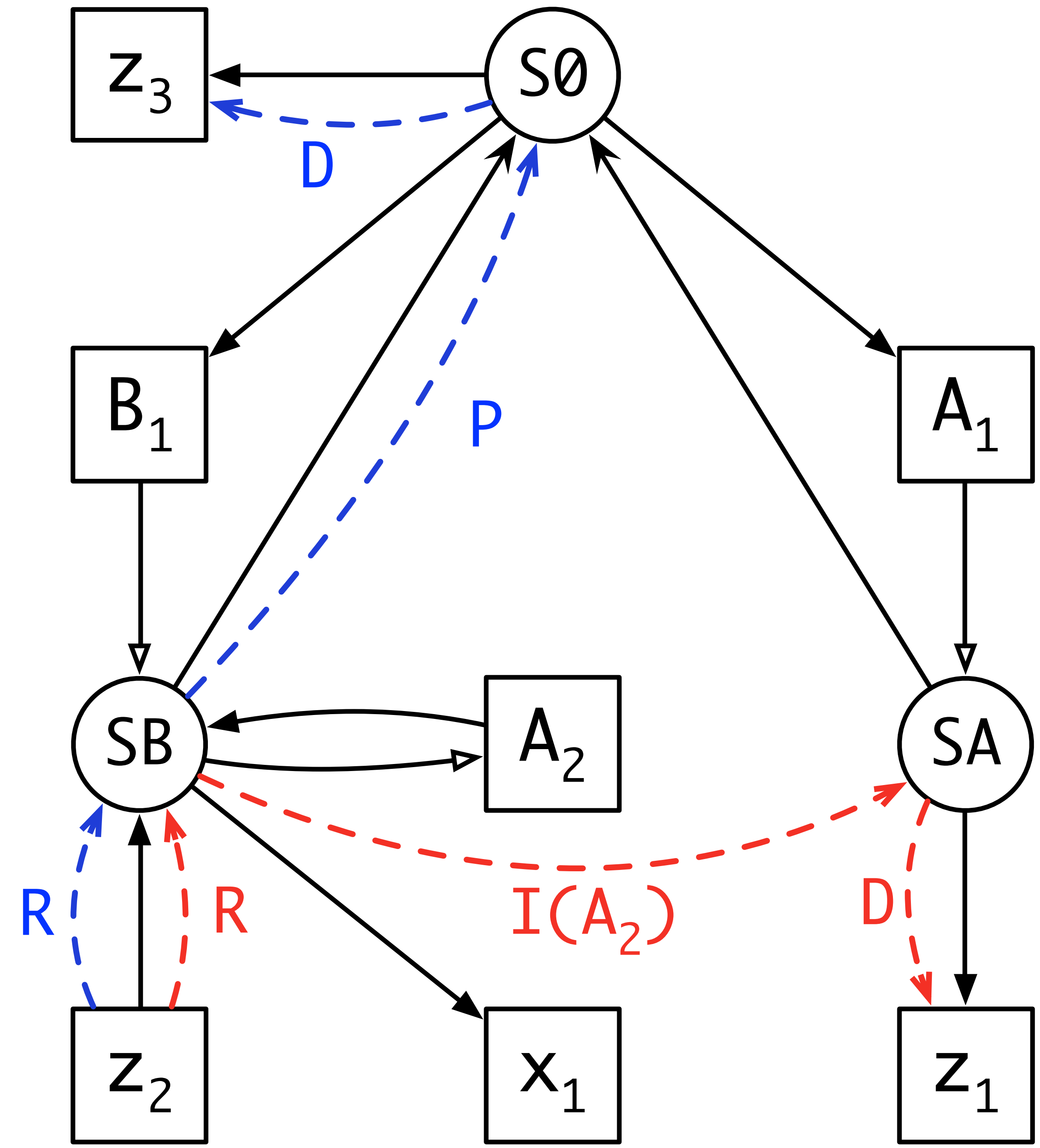
}



Imports shadow Parents

```
def z3 = 2                                S0

module A1 {
    def z1 = 5                            SA
}
module B1 {
    import A2                             SB
    def x1 = 1 + z2
}
```

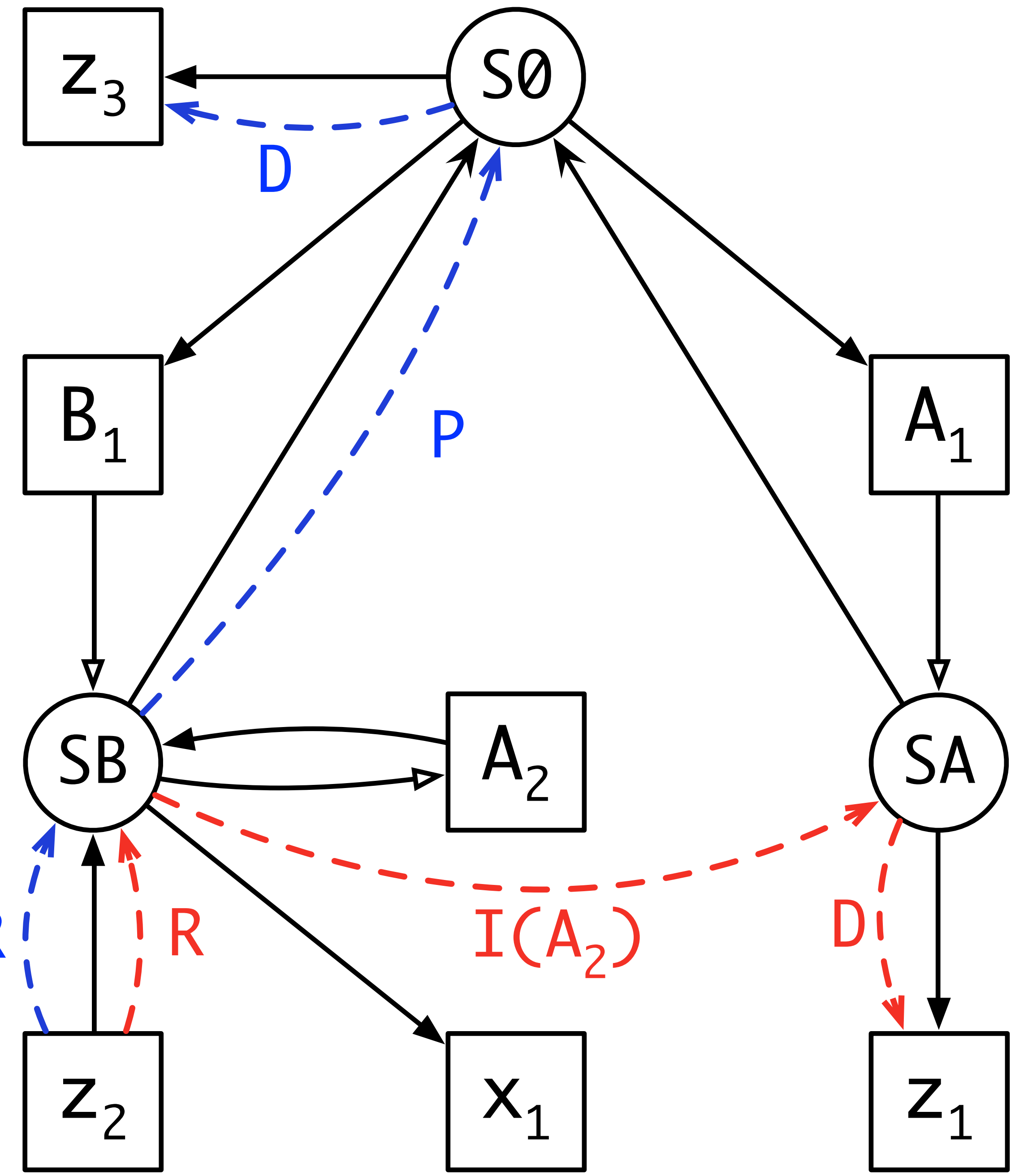


I(_).p' < P.p

Imports shadow Parents

```
def z3 = 2                S0

module A1 {
    def z1 = 5            SA
}
module B1 {
    import A2             SB
    def x1 = 1 + z2
}
```



$$\frac{}{I(\_).p' < P.p} \quad \Rightarrow \quad R.I(A_2).D < R.P.D$$

Imports vs. Includes

S0

$\texttt{def } z_3 = 2$

$\texttt{module } A_1 \ \{$

$\quad \texttt{def } z_1 = 5$ SA

$\}$

$\texttt{import } A_2$

$\texttt{def } x_1 = 1 + z_2$

Imports vs. Includes

S0

def z_3 = 2

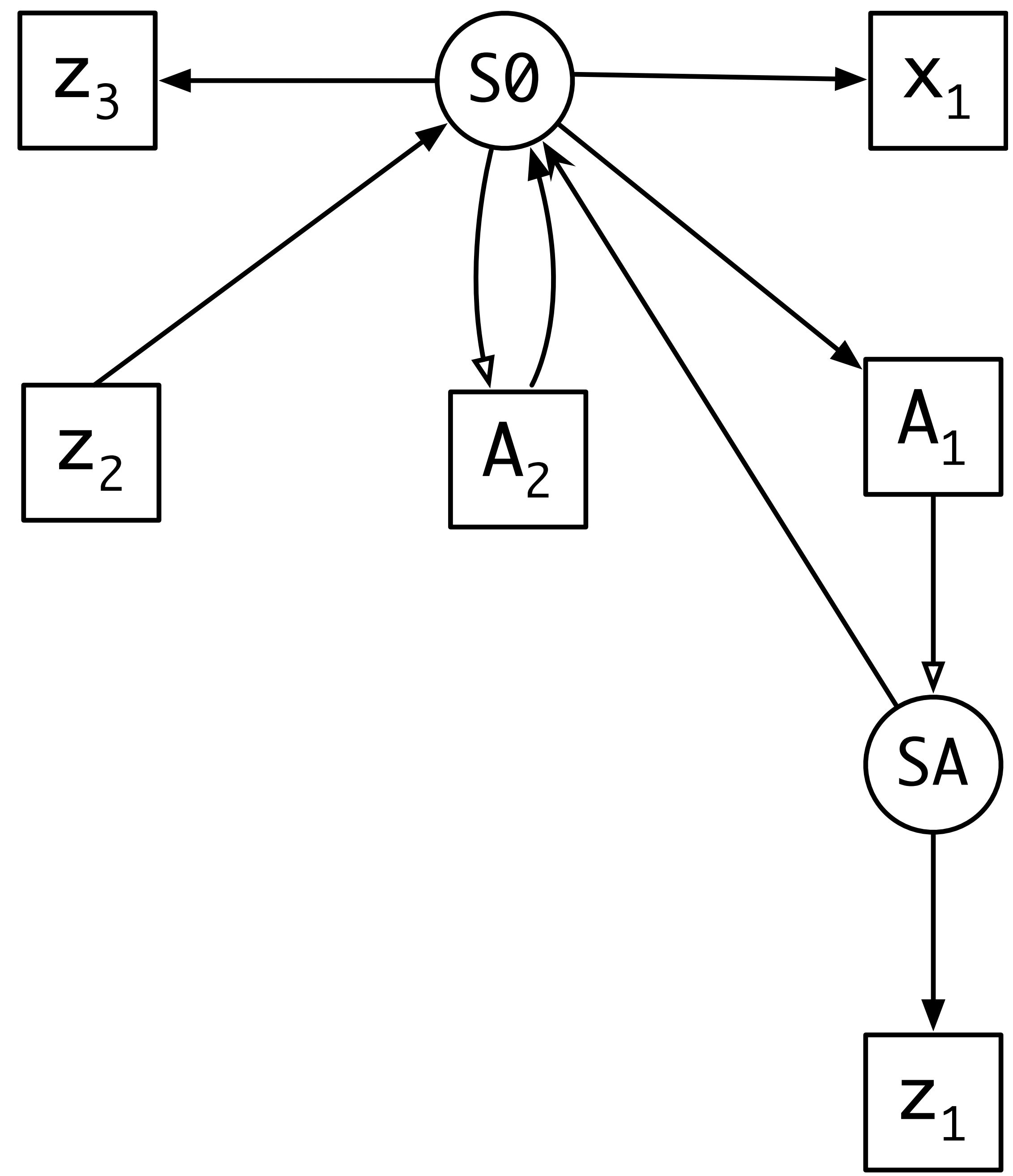
module A_1 {

def z_1 = 5 SA

}

import A_2

def x_1 = 1 + z_2



Imports vs. Includes

S0

$\texttt{def}\ z_3 = 2$

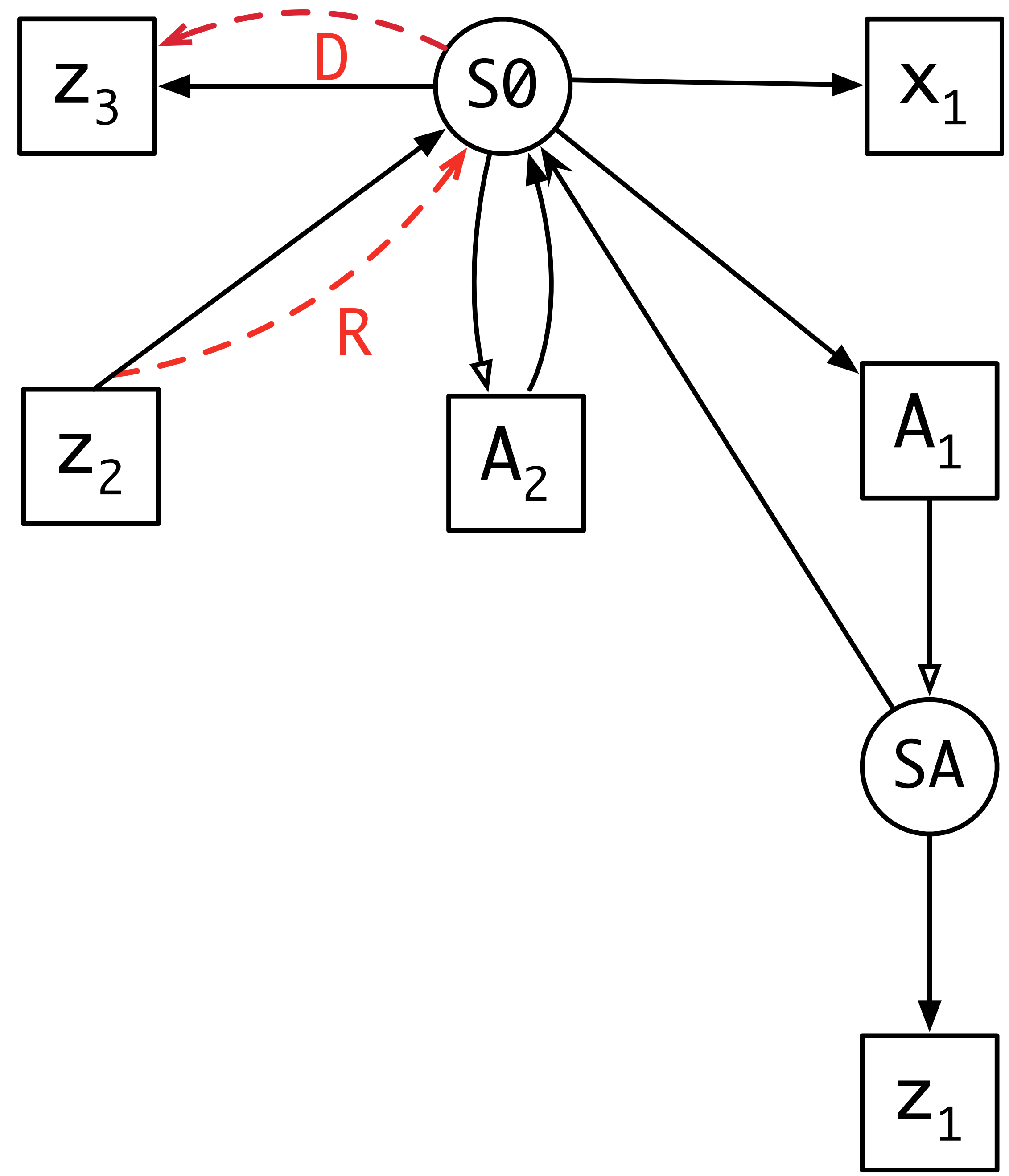
$\texttt{module}\ A_1\ \{$

$\quad \texttt{def}\ z_1 = 5$ SA

$\}$

$\texttt{import}\ A_2$

$\texttt{def}\ x_1 = 1 + z_2$



Imports vs. Includes

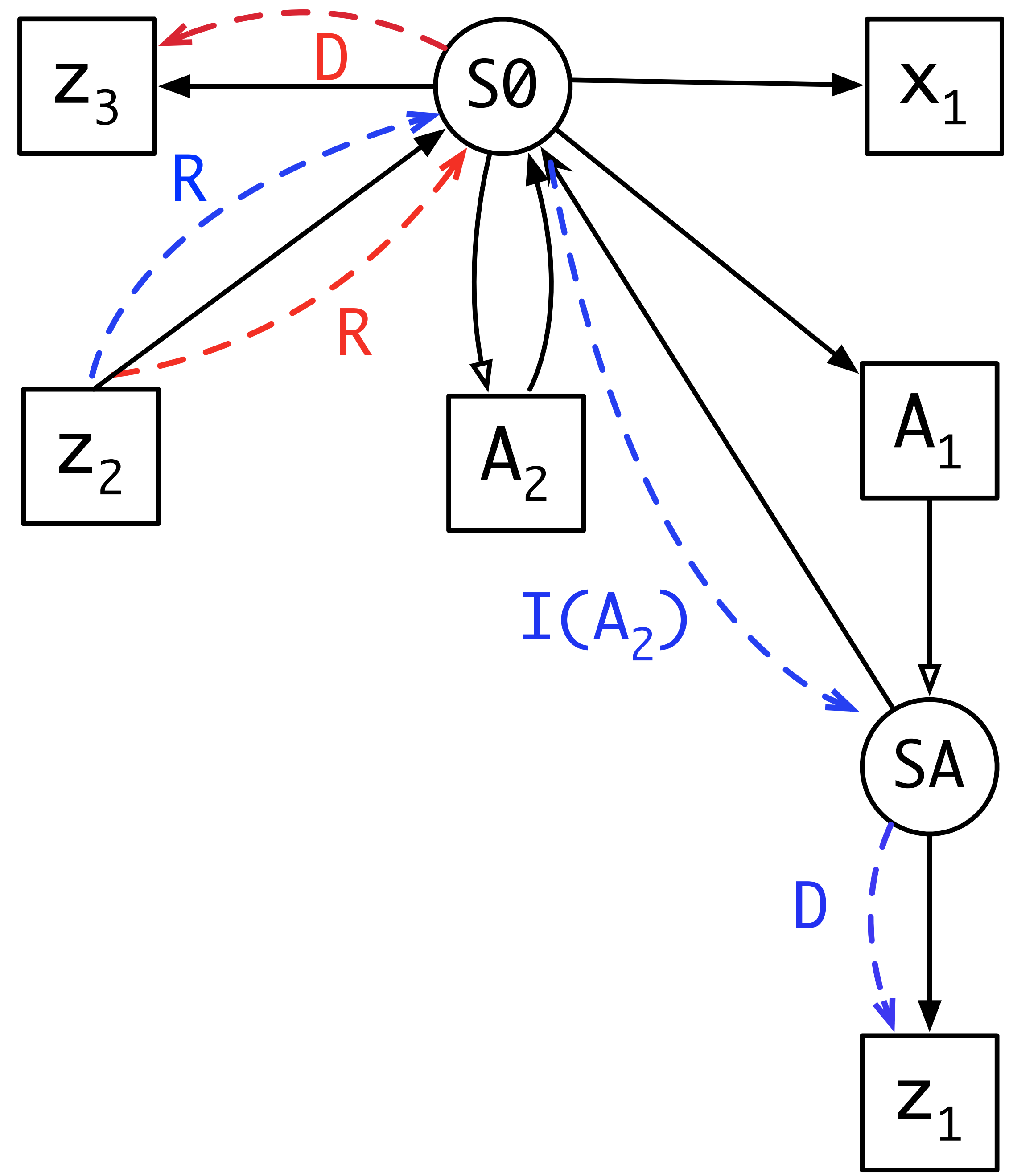
S0

```
def z₃ = 2

module A₁ {
  def z₁ = 5
}

import A₂
def x₁ = 1 + z₂
```

SA



Imports vs. Includes

S0

def $z_3 = 2$

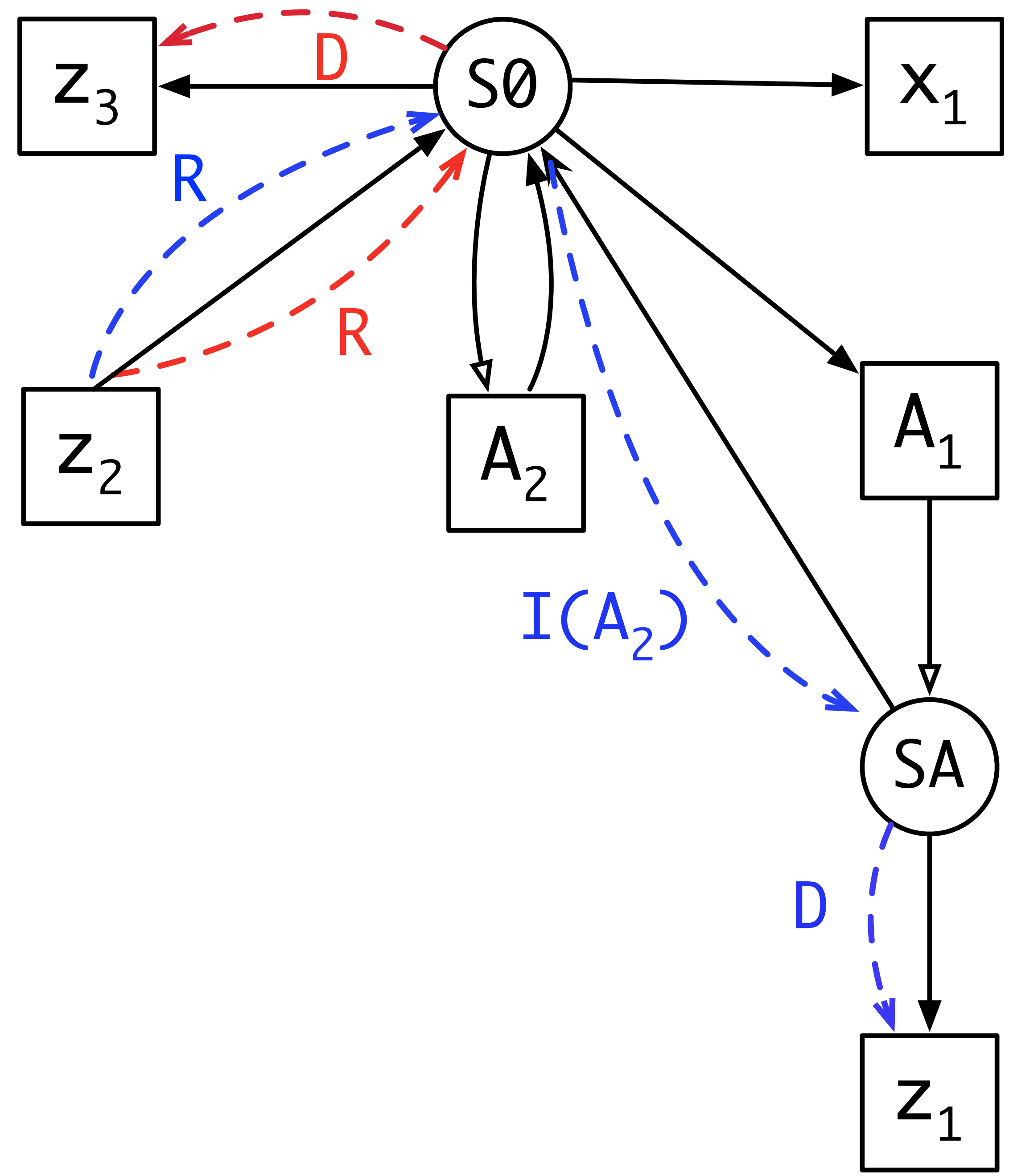
module A_1 {

def $z_1 = 5$ SA

}

import A_2

def $x_1 = 1 + z_2$



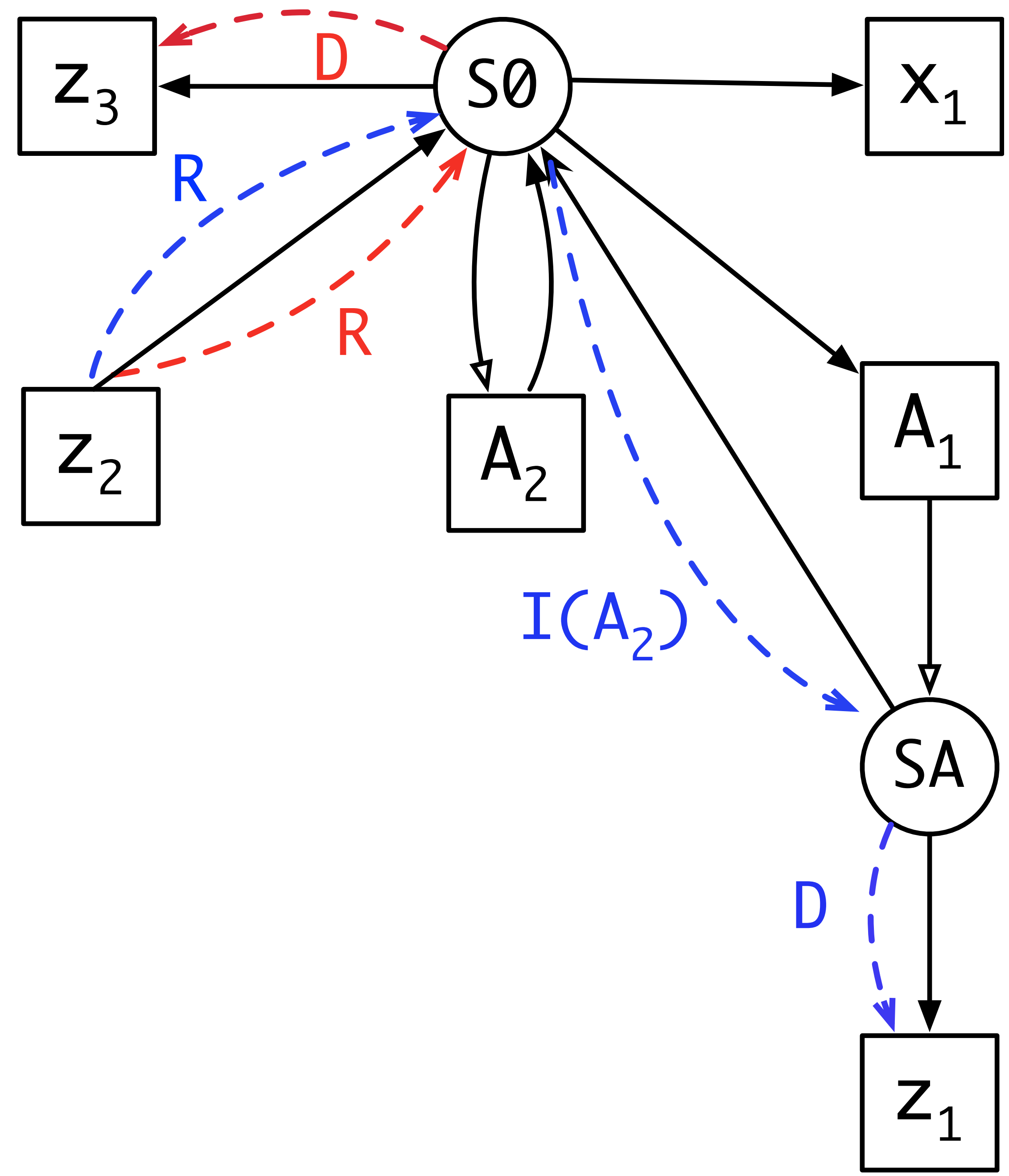
D < I(_).p’

Imports vs. Includes

```
def z₃ = 2                      S0

module A₁ {
    def z₁ = 5                  SA
}

import A₂
def x₁ = 1 + z₂
```



$$\frac{}{\color{red}{D} < \color{blue}{I(\_).p'}} \quad \Rightarrow \quad \color{red}{R.D} < \color{blue}{R.I(A_2).D}$$

Imports vs. Includes

S0

$\mathtt{def}\ z_3 = 2$

$\mathtt{module}\ A_1\ \{$

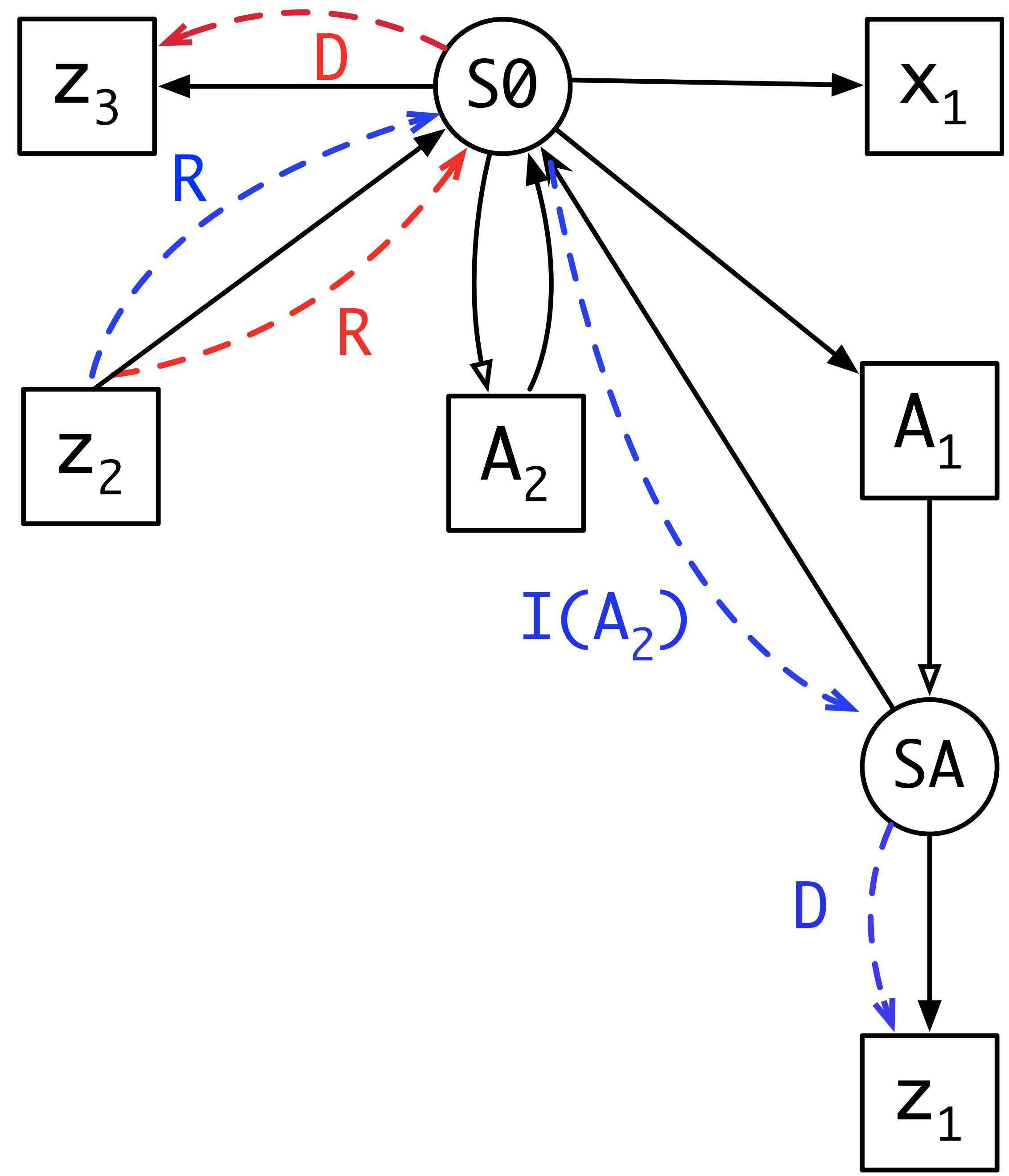
SA

$\quad \mathtt{def}\ z_1 = 5$

$\}$

$\mathtt{include}\ A_2$

$\mathtt{def}\ x_1 = 1 + z_2$



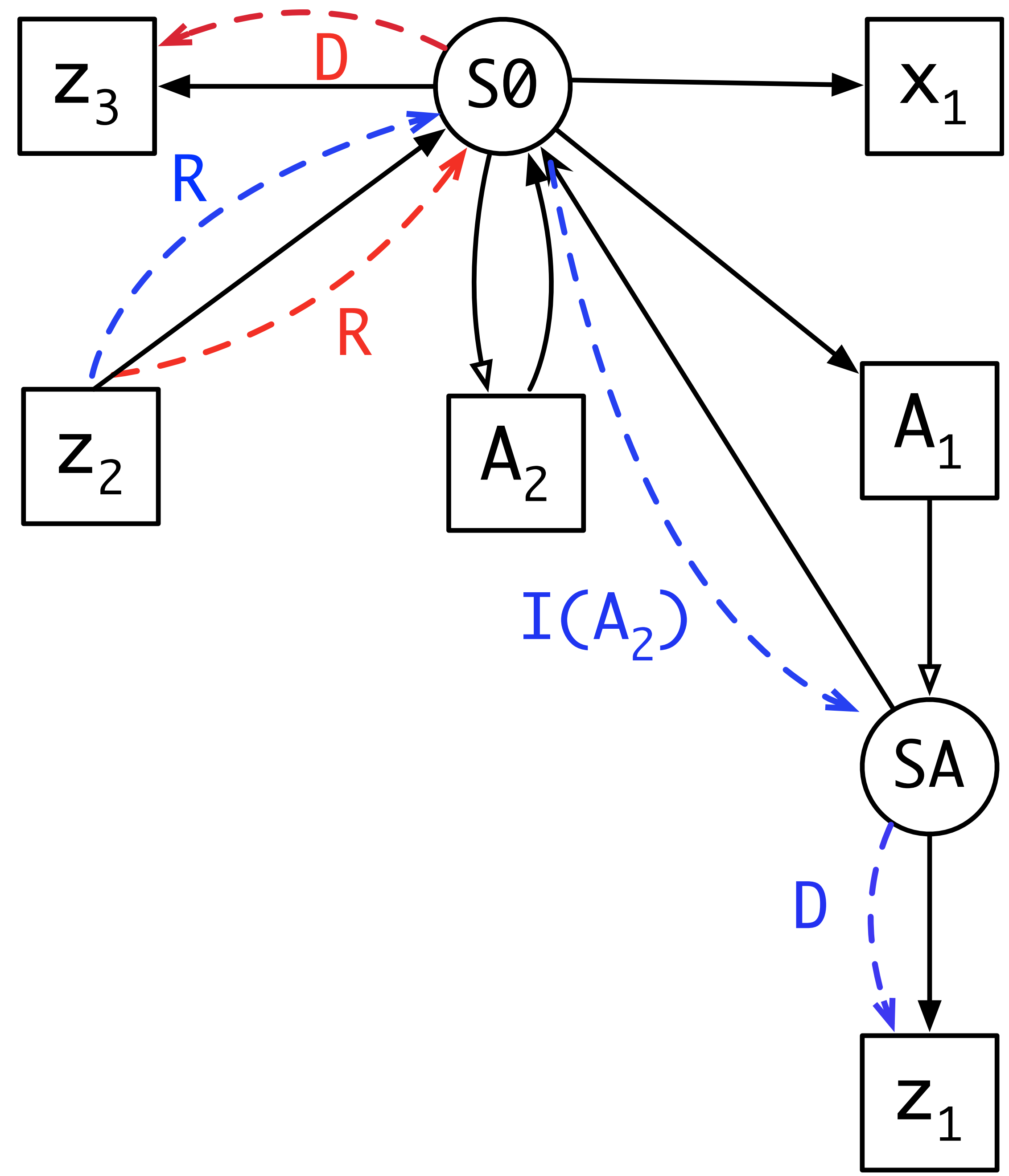
$$\frac{}{D < I(\_).p'} \quad \Rightarrow \quad R.D < R.I(A_2).D$$

Imports vs. Includes

```
def z₃ = 2                 S0

module A₁ {
  def z₁ = 5               SA
}

include A₂
def x₁ = 1 + z₂
```



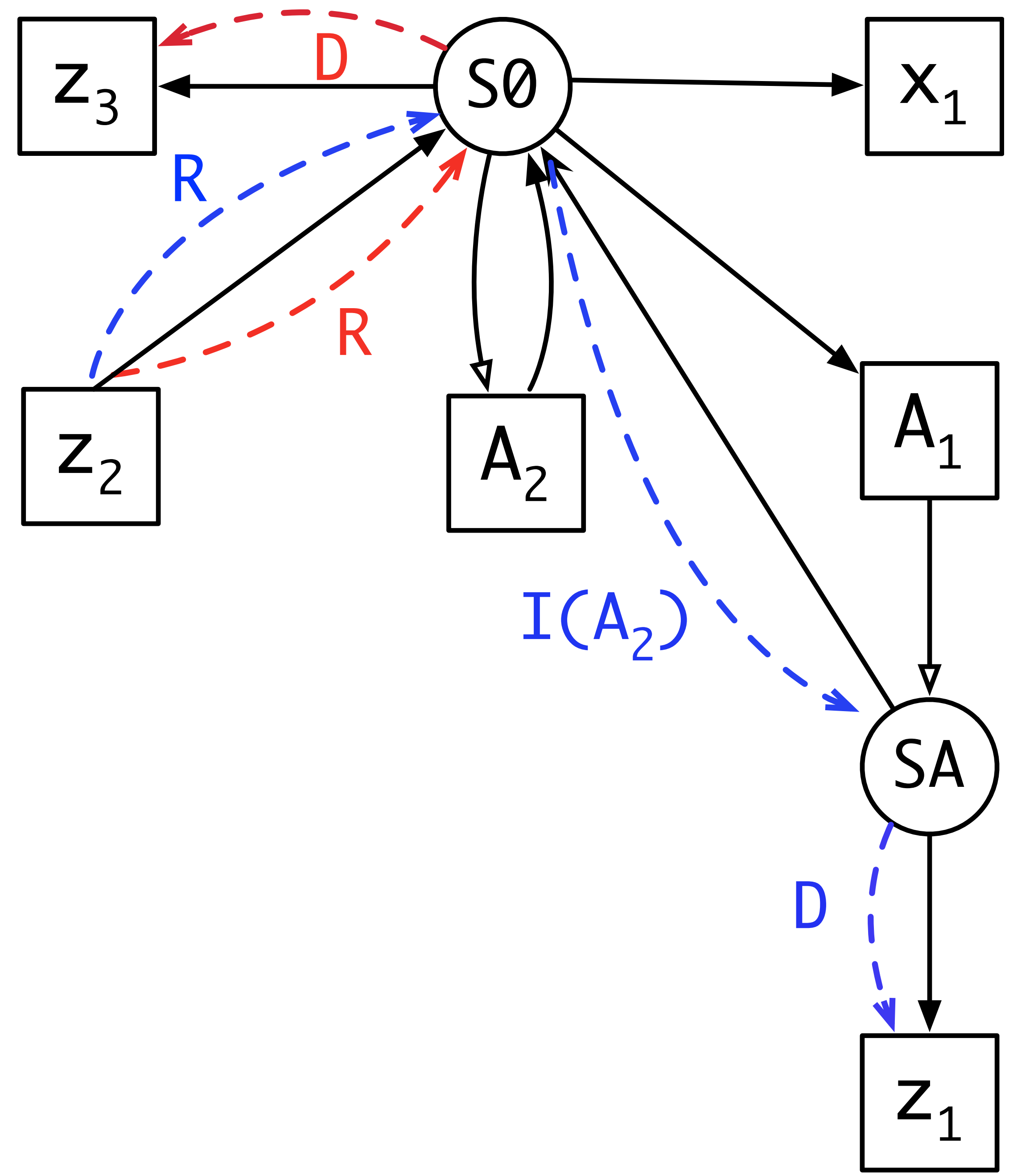
$$\cancel{\frac{\quad}{D < I(\_).p'}} \quad \Rightarrow \quad R.D < R.I(A_2).D$$

Imports vs. Includes

```
def z3 = 2                S0

module A1 {
    def z1 = 5            SA
}

include A2
def x1 = 1 + z2
```



$$\cancel{\dfrac{}{D < I(\_).p'}}$$

Qualified Names

S0

```
module N₁ {
    def s₁ = 5
}

module M₁ {
    def x₁ = 1 + N₂.s₂
}
```

Qualified Names

S0

$\texttt{module}\ N_1\ \{$

$\quad \texttt{def}\ s_1 = 5$

$\}$

$\texttt{module}\ M_1\ \{$

$\quad \texttt{def}\ x_1 = 1 + N_2.s_2$

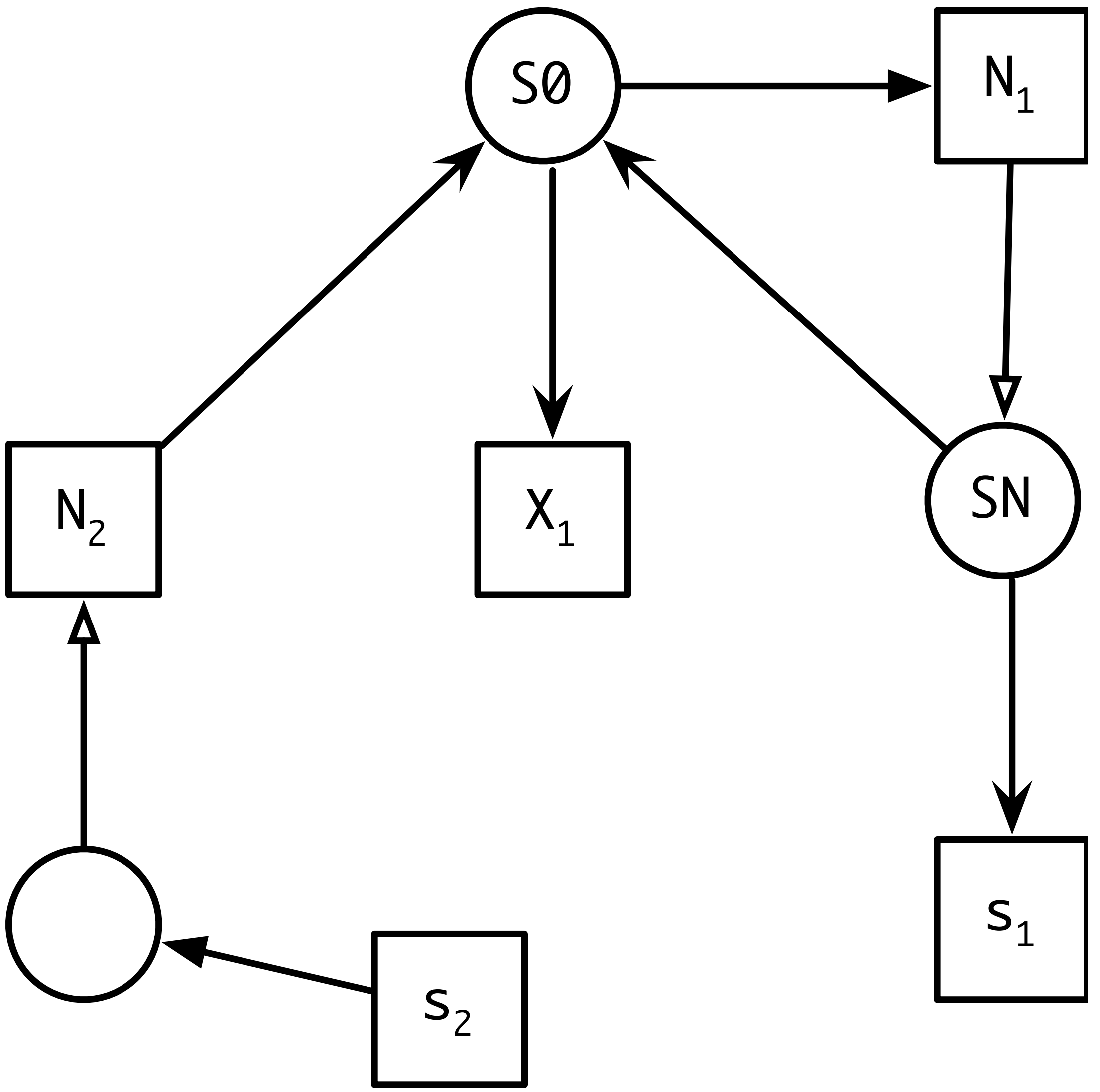
$\}$

Qualified Names

S0

```
module N₁ {
    def s₁ = 5
}

module M₁ {
    def x₁ = 1 + N₂.s₂
}
```

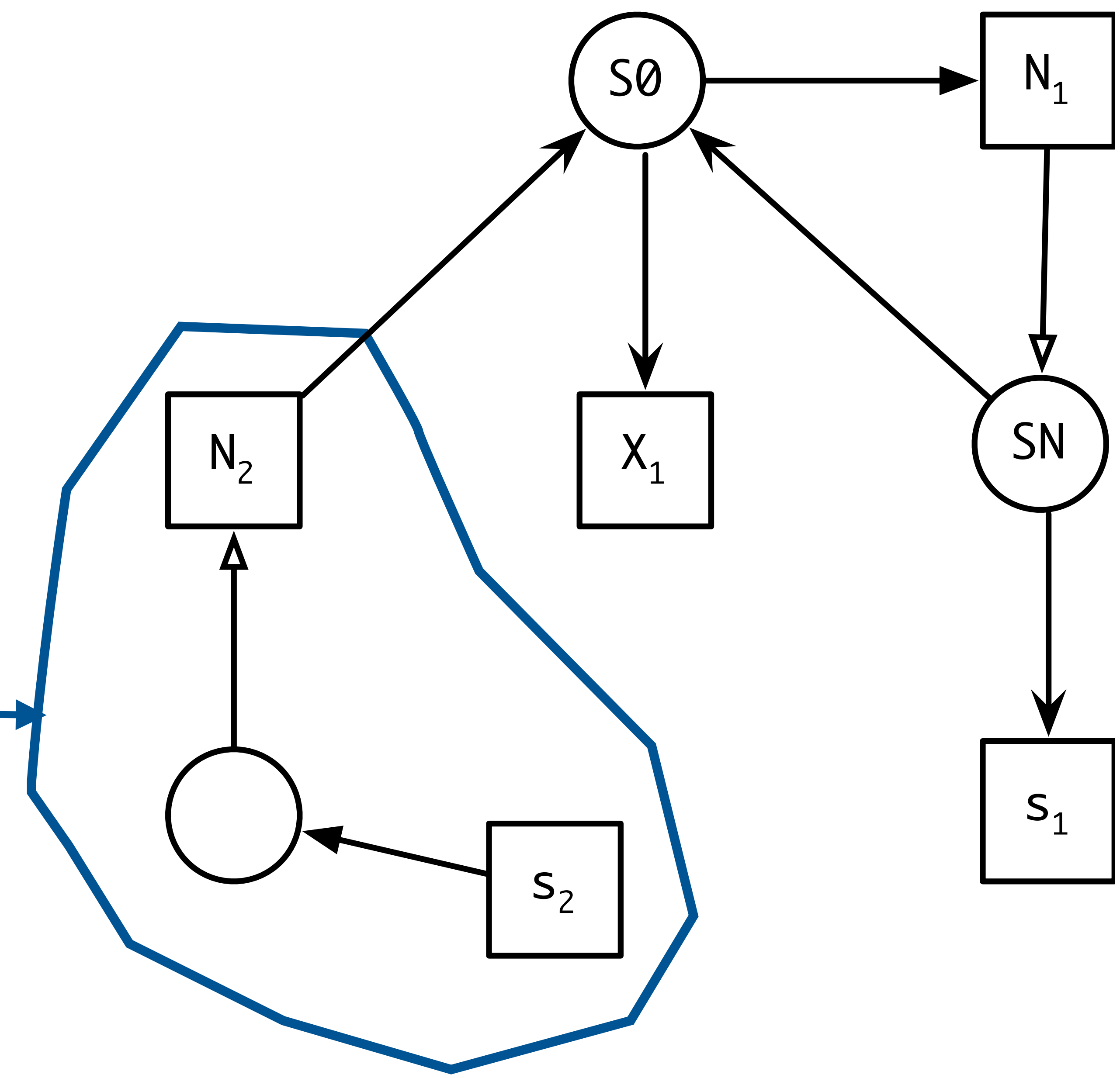


Qualified Names

S0

```
module N₁ {
    def s₁ = 5
}

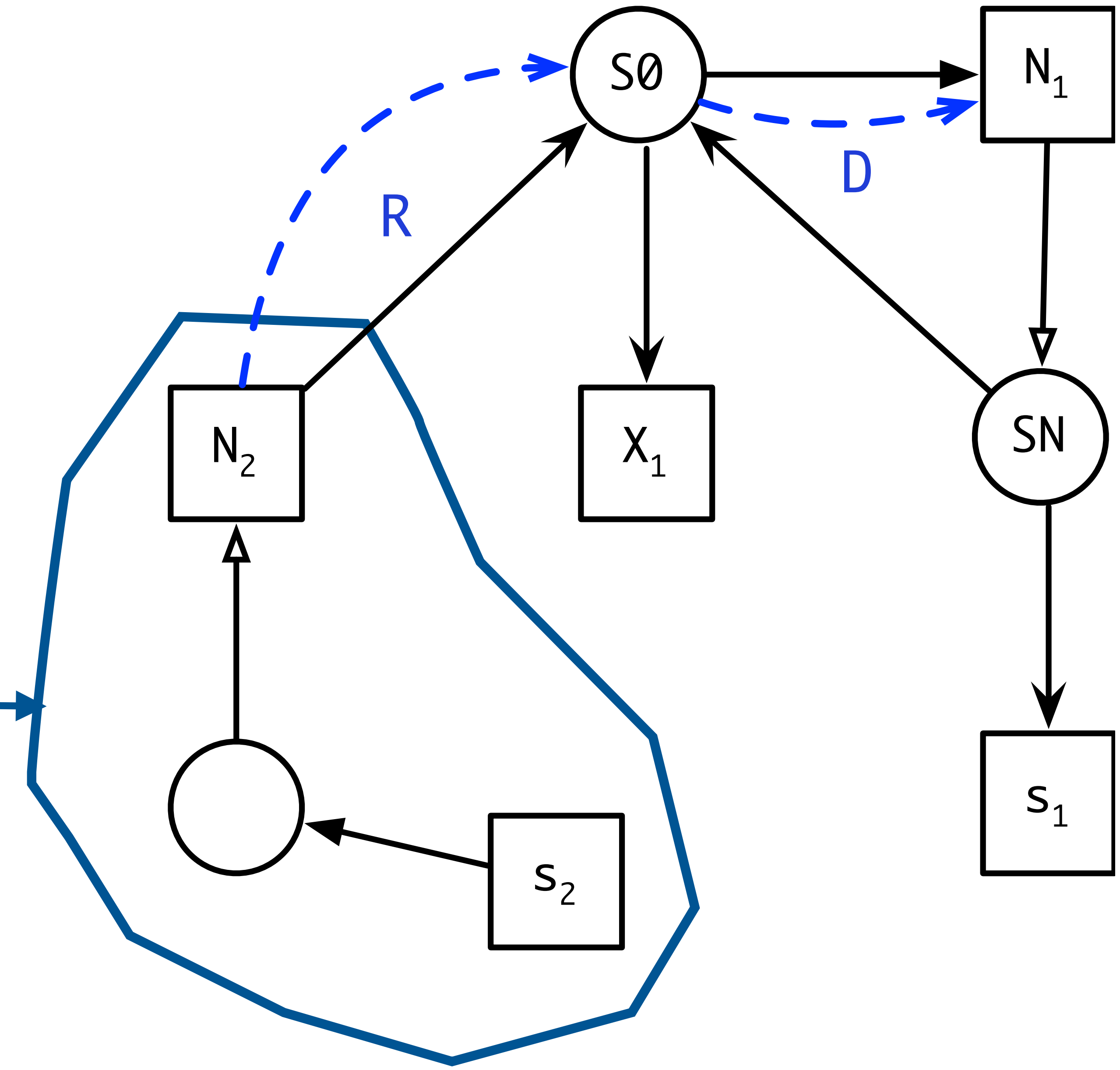
module M₁ {
    def x₁ = 1 + N₂.s₂
}
```



Qualified Names

```
module N₁ {
    def s₁ = 5
}

module M₁ {
    def x₁ = 1 + N₂.s₂
}
```

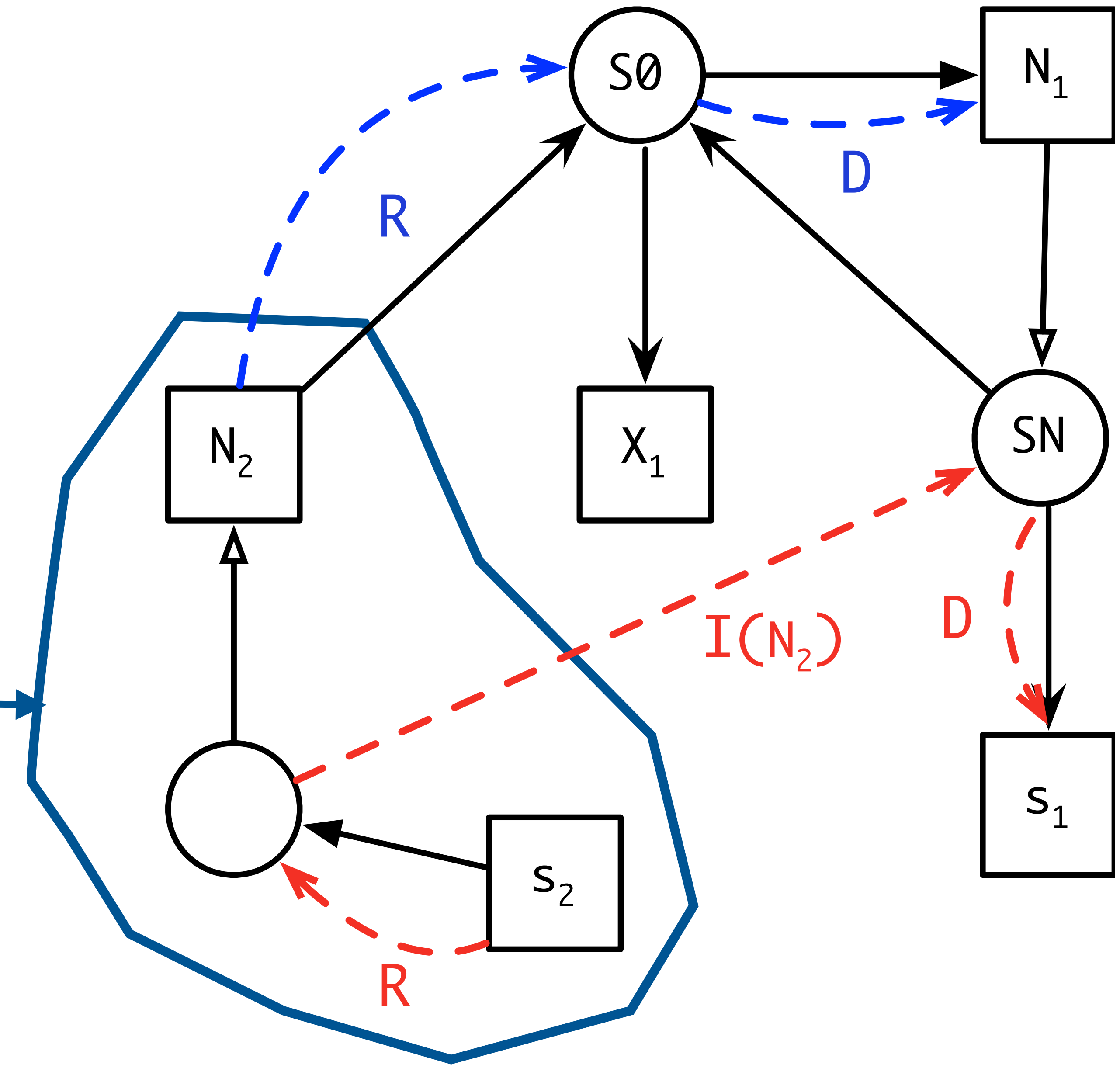


Qualified Names

S0

```
module N1 {
    def s1 = 5
}

module M1 {
    def x1 = 1 + N2.s2
}
```



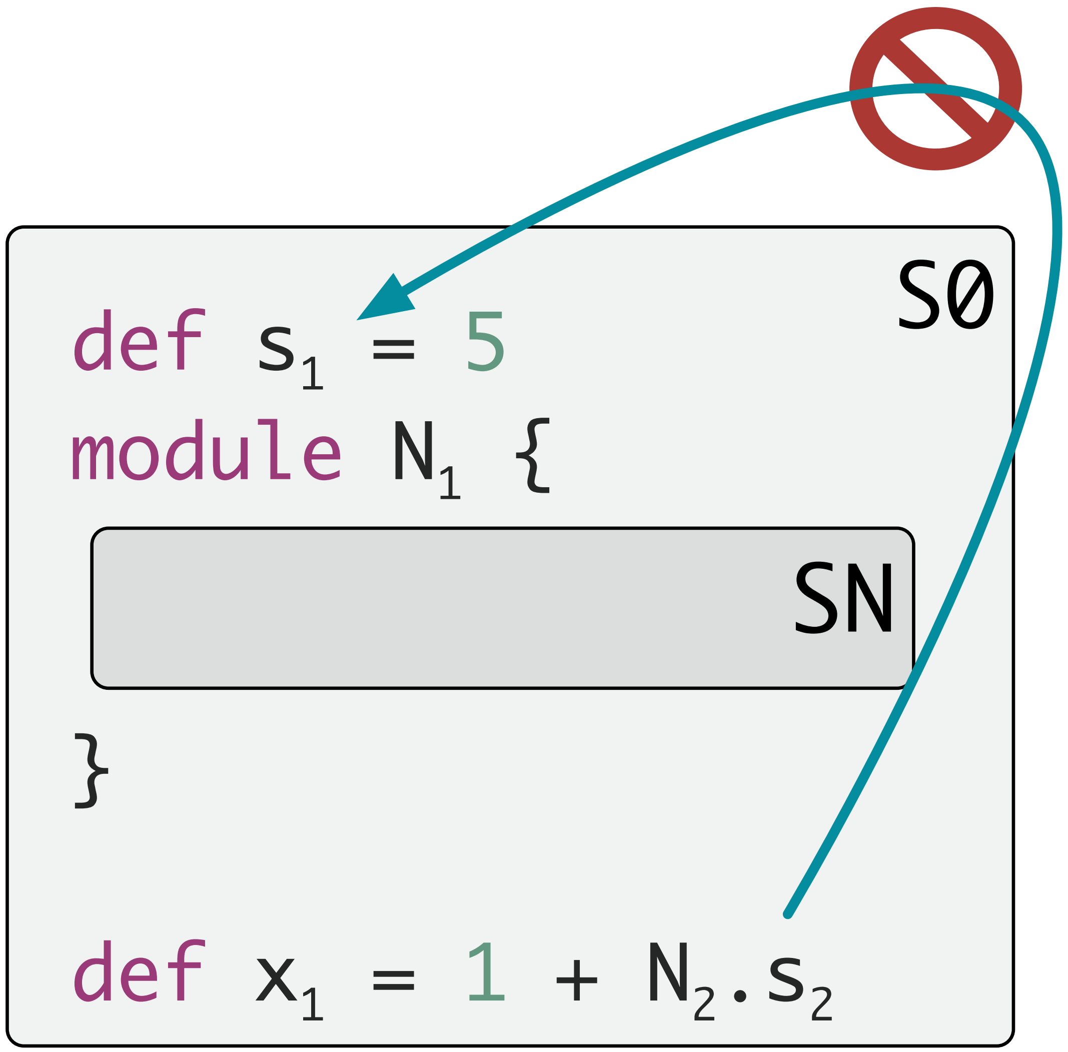
Import Parents

S0

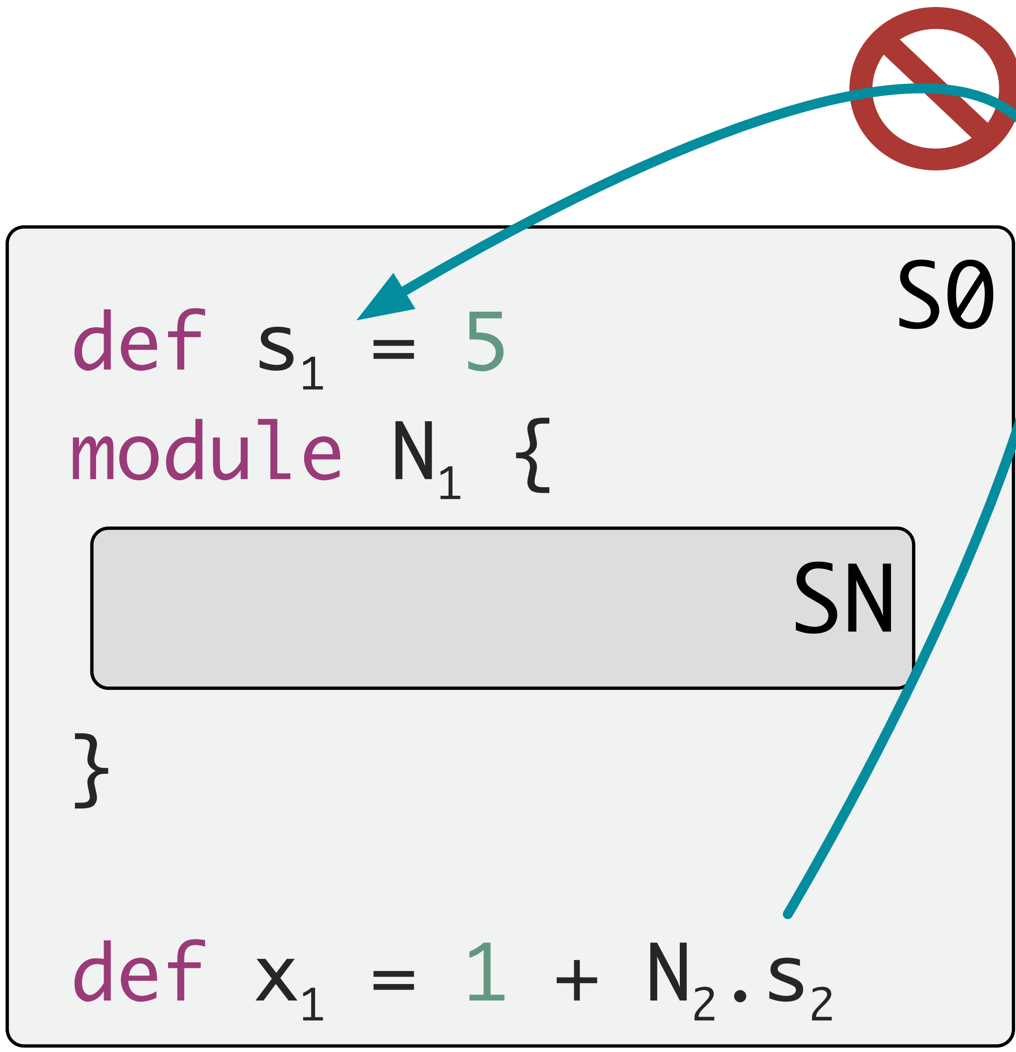
```
def s₁ = 5
module N₁ {
    SN
}

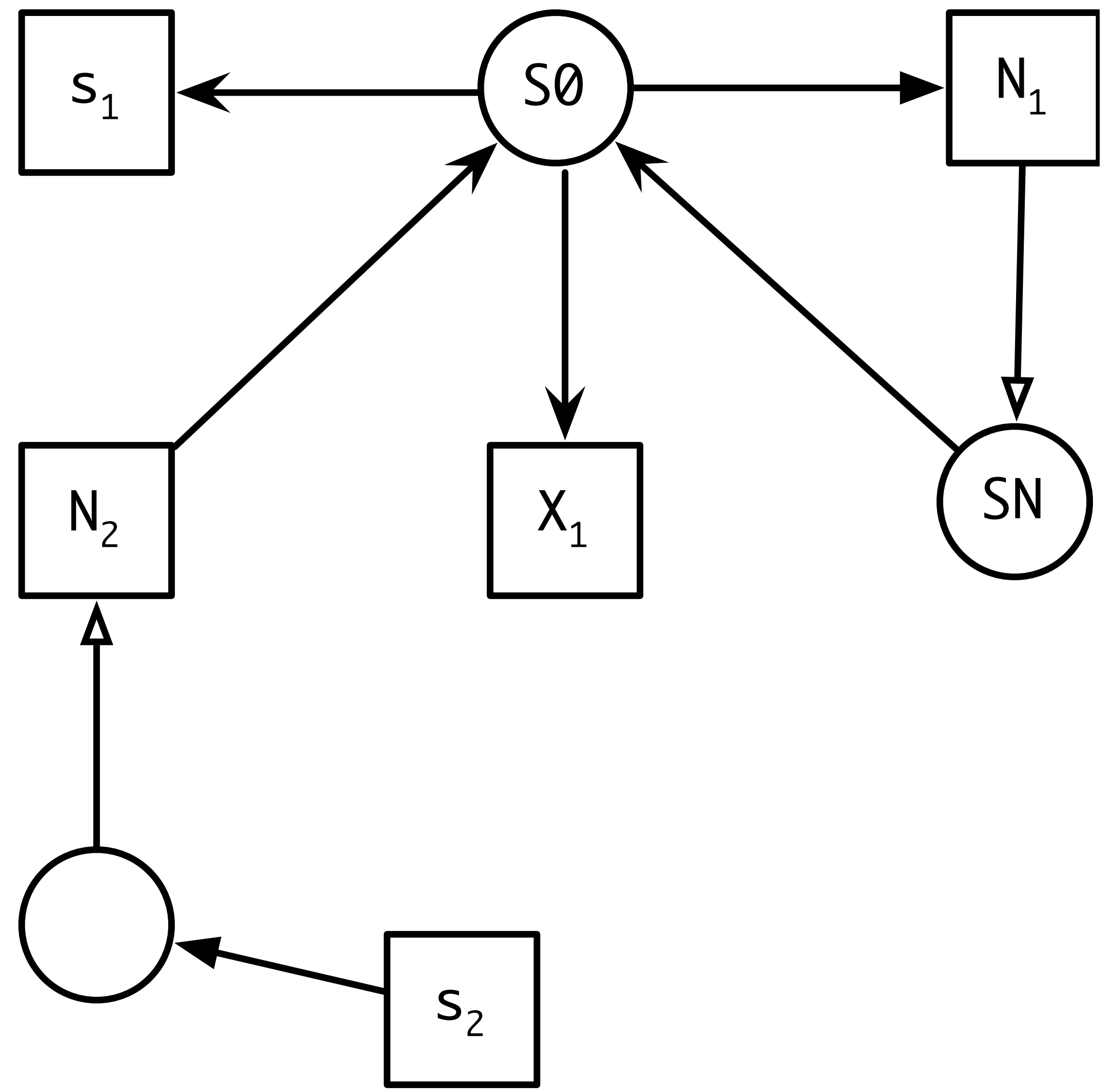
def x₁ = 1 + N₂.s₂
```

Import Parents

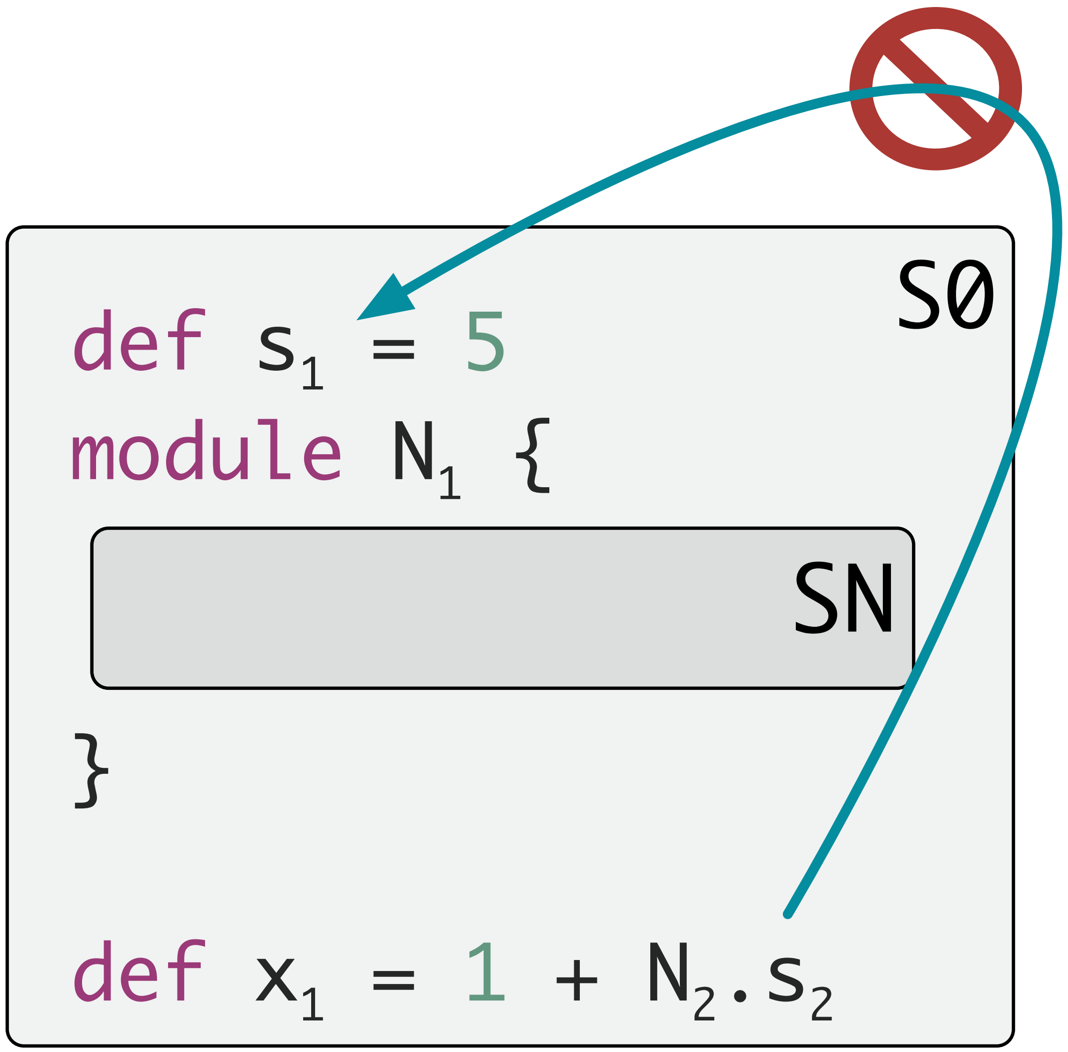


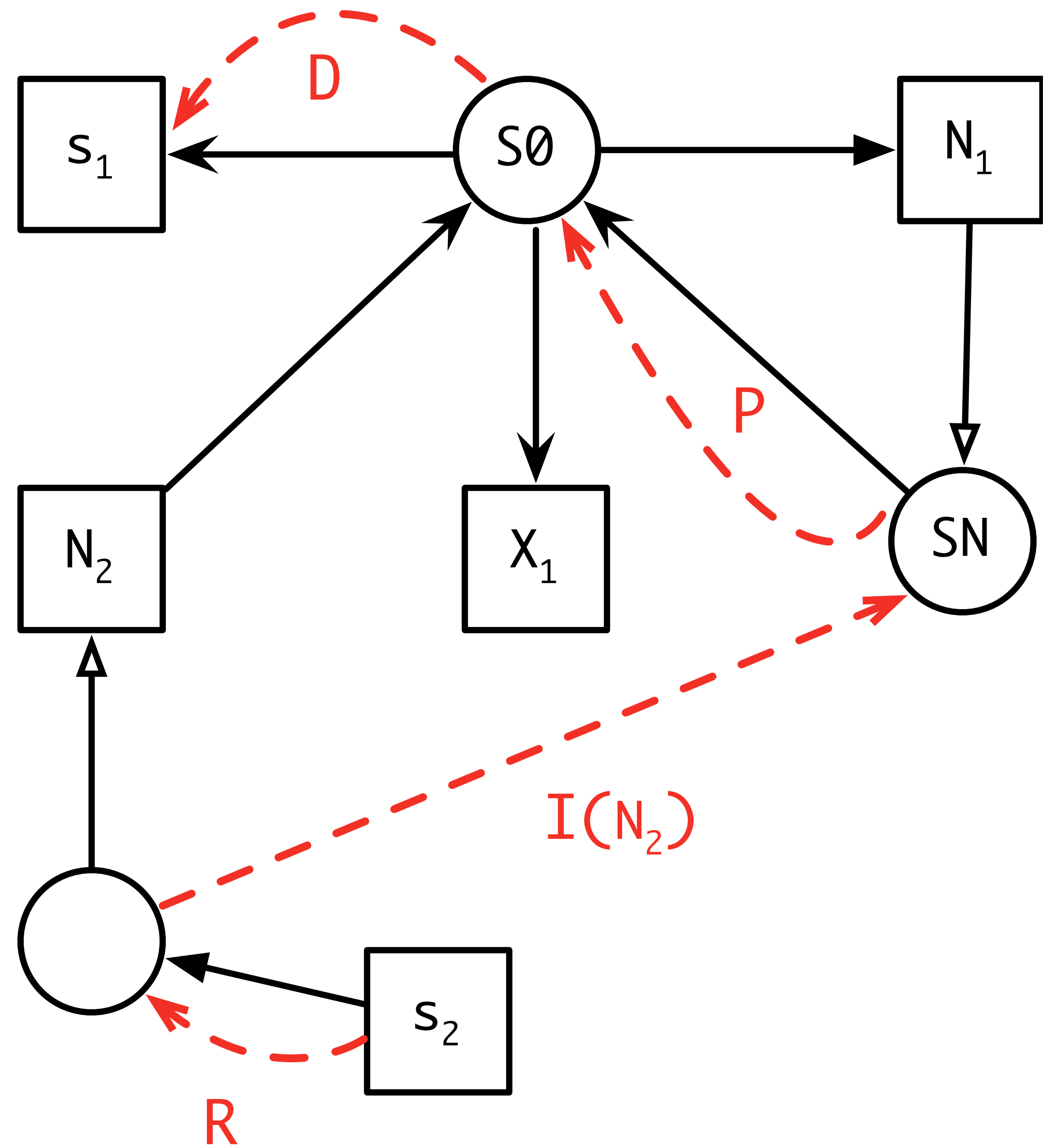
Import Parents



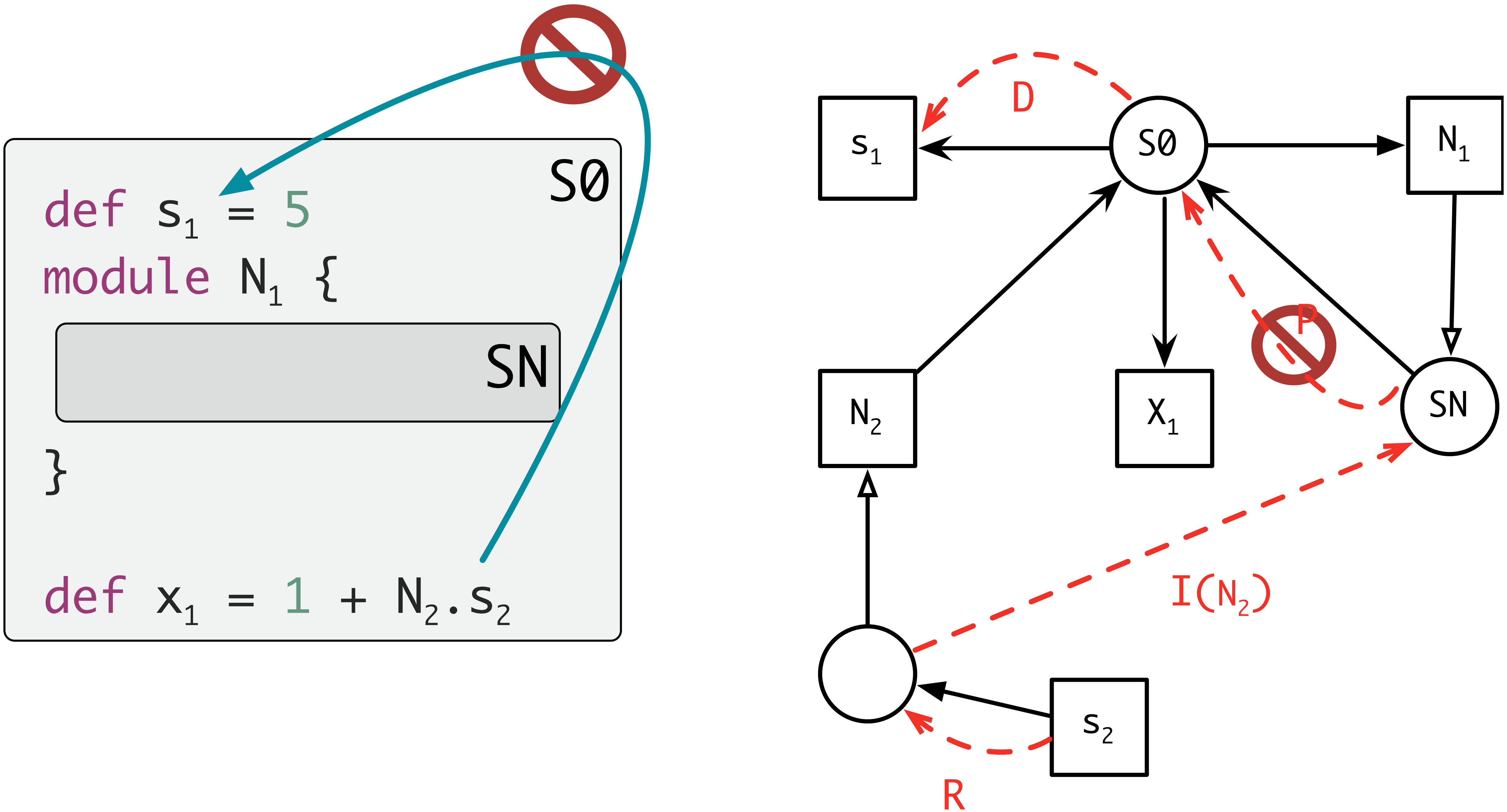


Import Parents

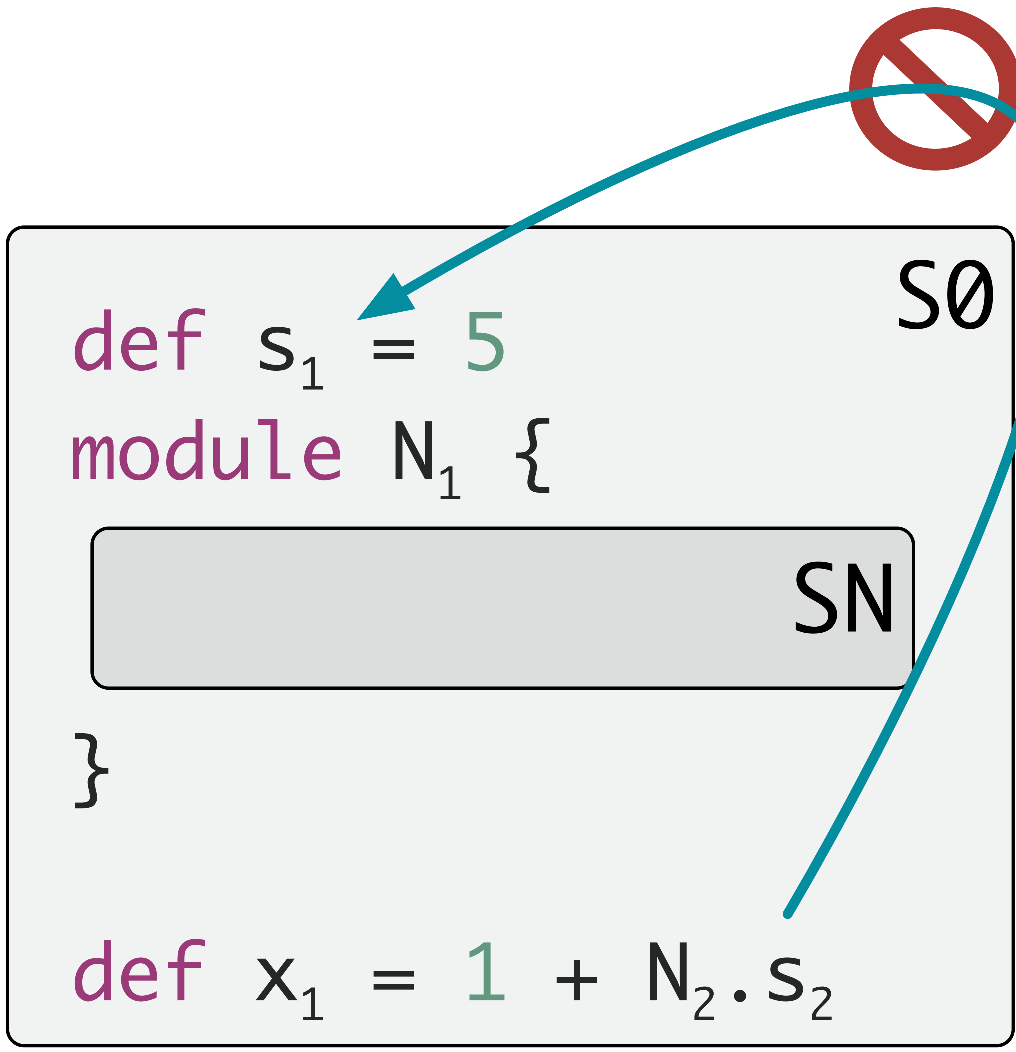


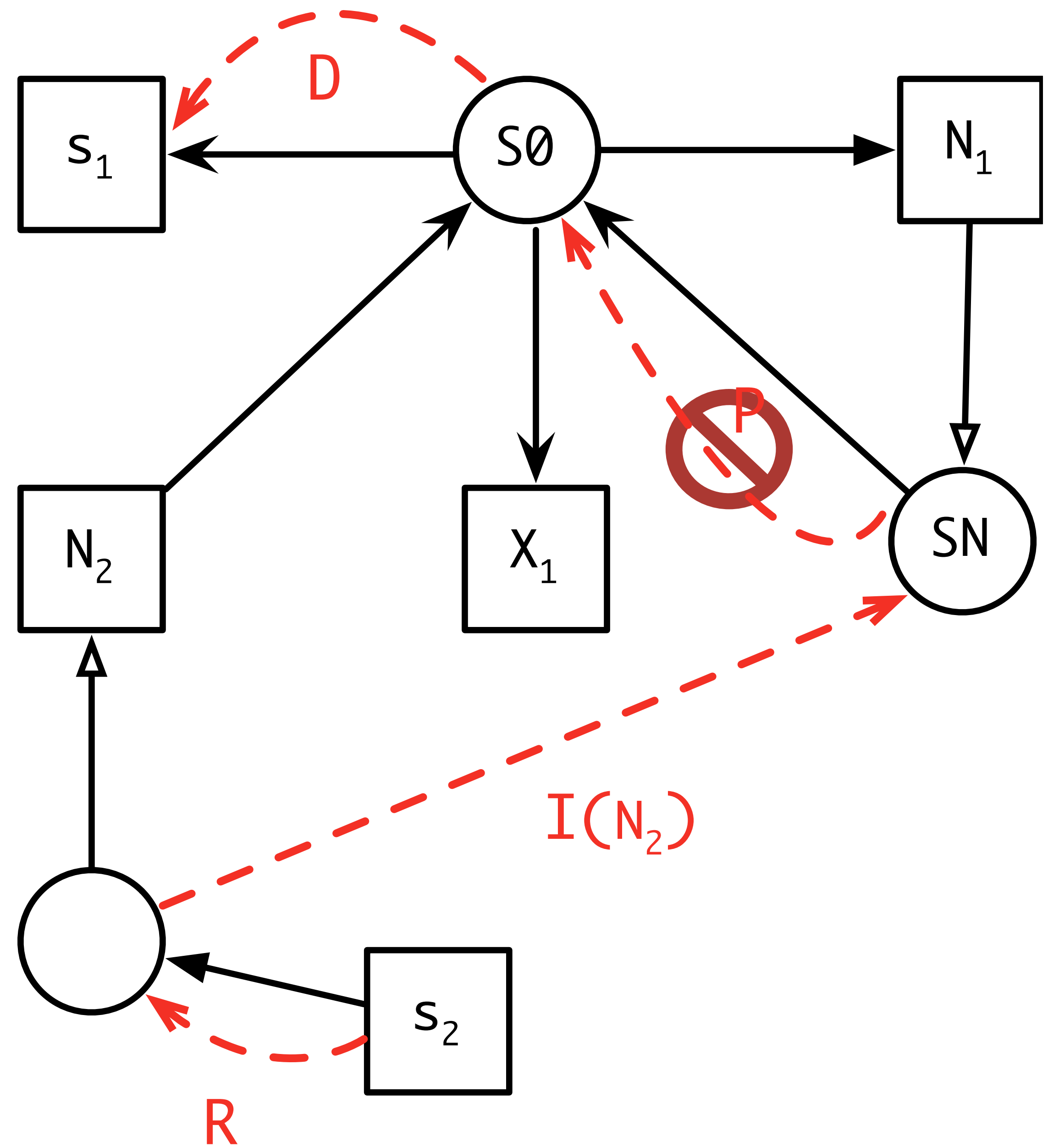


Import Parents



Import Parents





Well formed path: R.P*.I(_)*.D

Transitive vs. Non-Transitive

```
module A₁ {
  def z₁ = 5            SA
}
module B₁ {
  import A₂             SB
}
module C₁ {
  import B₂             SC
  def x₁ = 1 + z₂
}
```

Transitive vs. Non-Transitive

module A_1 {

  def z_1 = 5 SA

}

module B_1 {

  import A_2 SB

}

module C_1 {

  import B_2 SC

  def x_1 = 1 + z_2

}

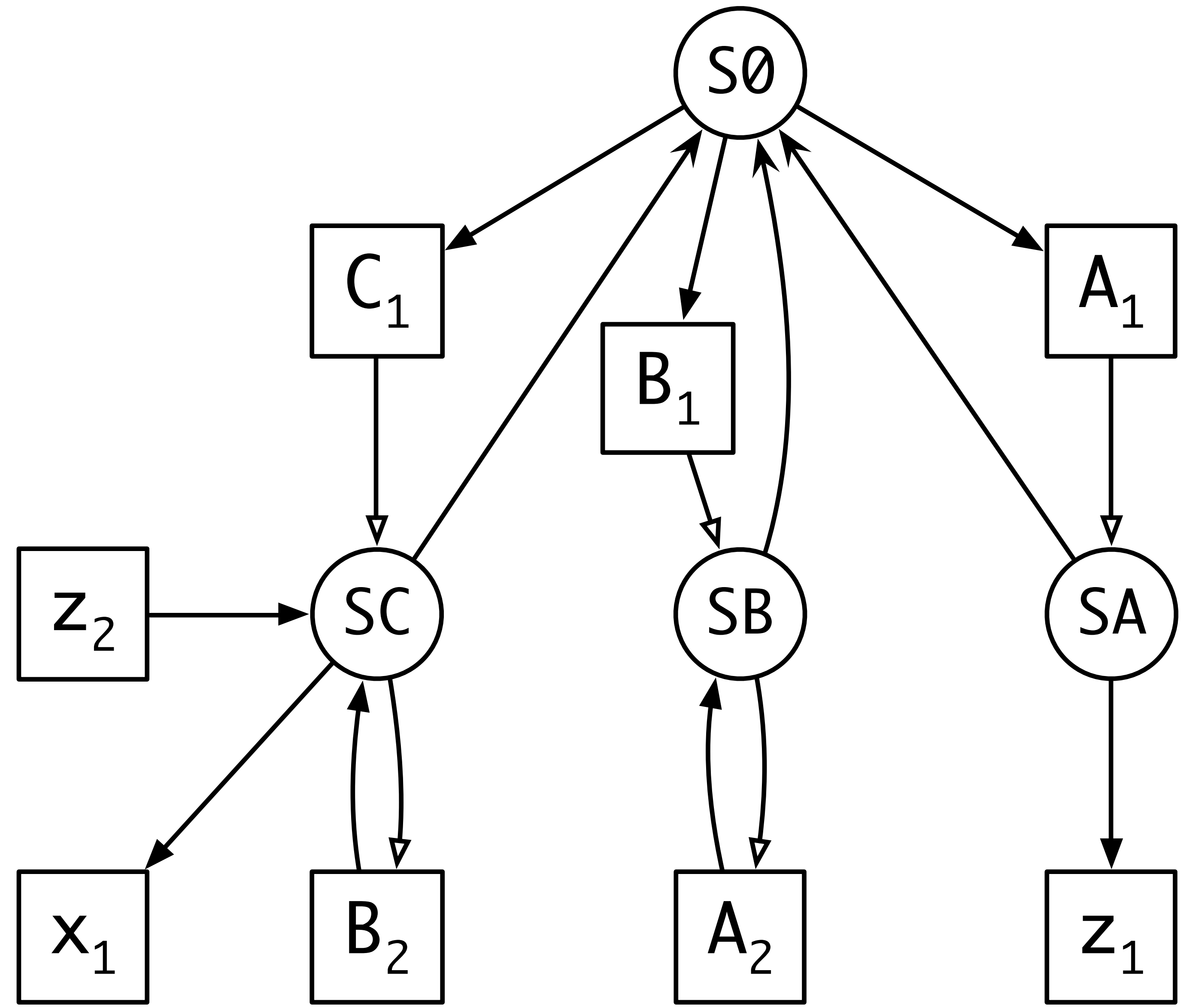
Transitive vs. Non-Transitive

```
module A₁ {
  def z₁ = 5              SA
}
module B₁ {
  import A₂               SB
}
module C₁ {
  import B₂               SC
  def x₁ = 1 + z₂
}
```

??

Transitive vs. Non-Transitive

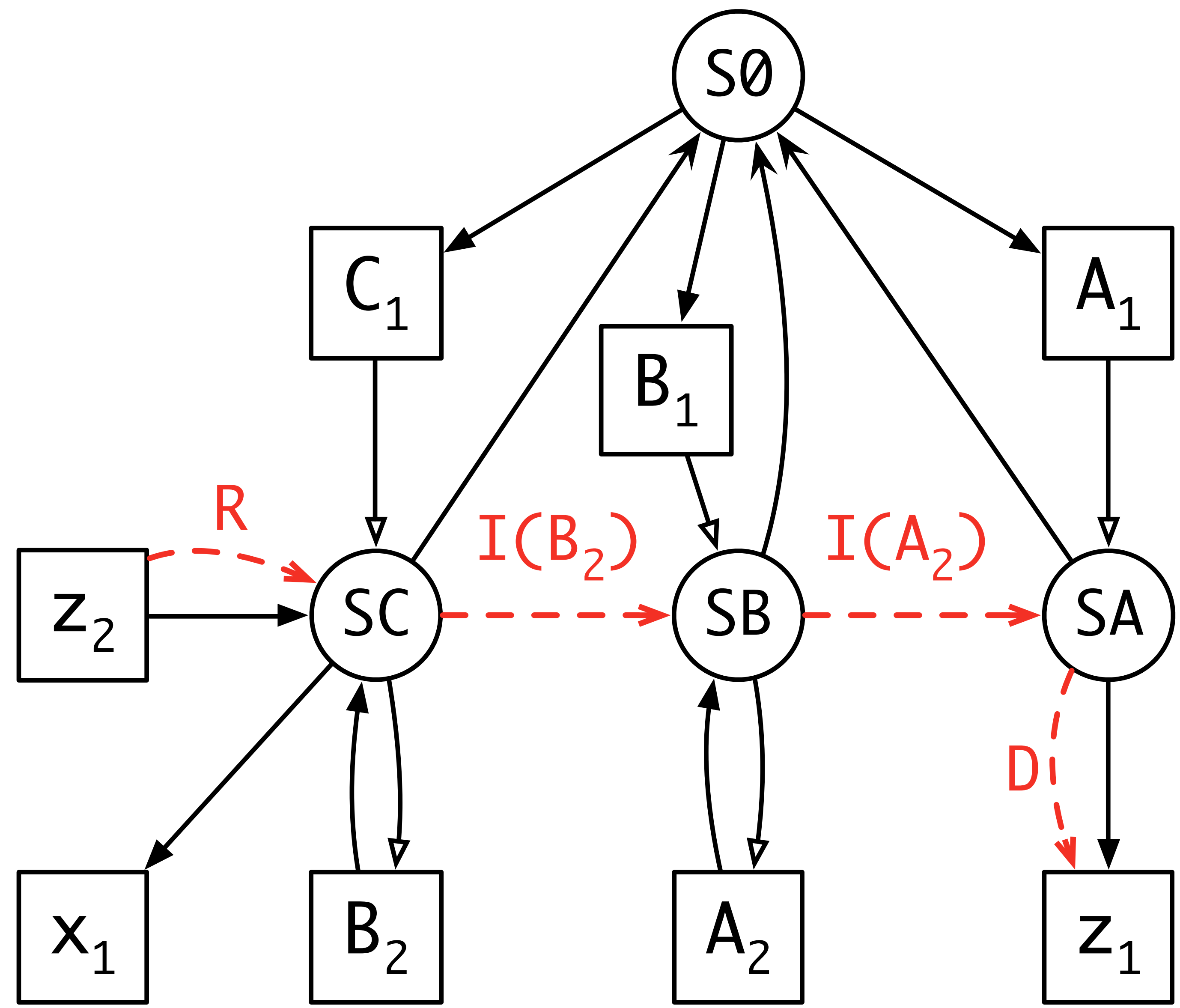
```
module A_1 {
    def z_1 = 5                  SA
}
                        ??
module B_1 {
    import A_2                   SB
}
module C_1 {
    import B_2                   SC
    def x_1 = 1 + z_2
}
```



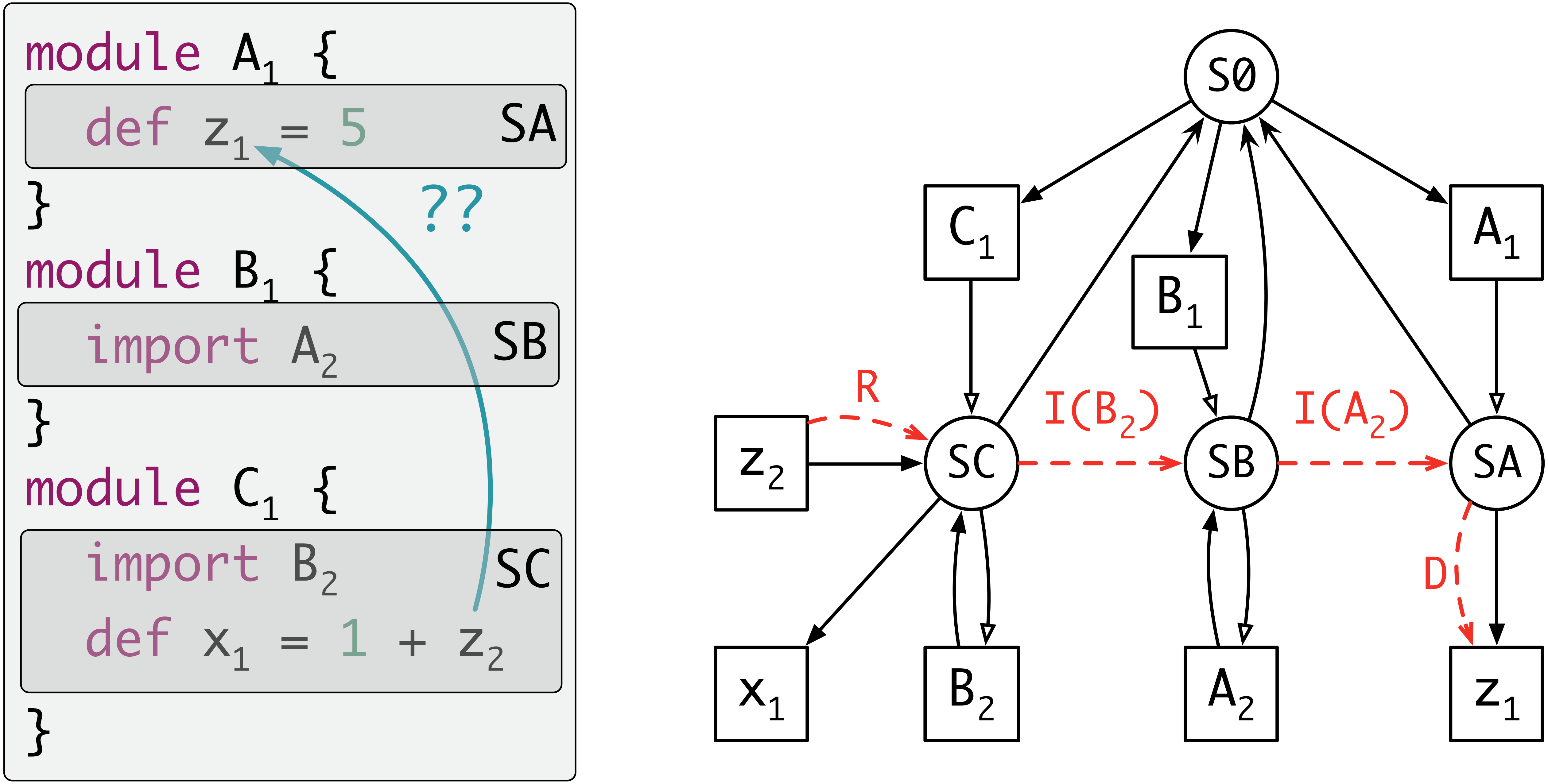
Transitive vs. Non-Transitive

```
module A₁ {
  def z₁ = 5          SA
}
module B₁ {
  import A₂           SB
}
module C₁ {
  import B₂           SC
  def x₁ = 1 + z₂
}
```

??



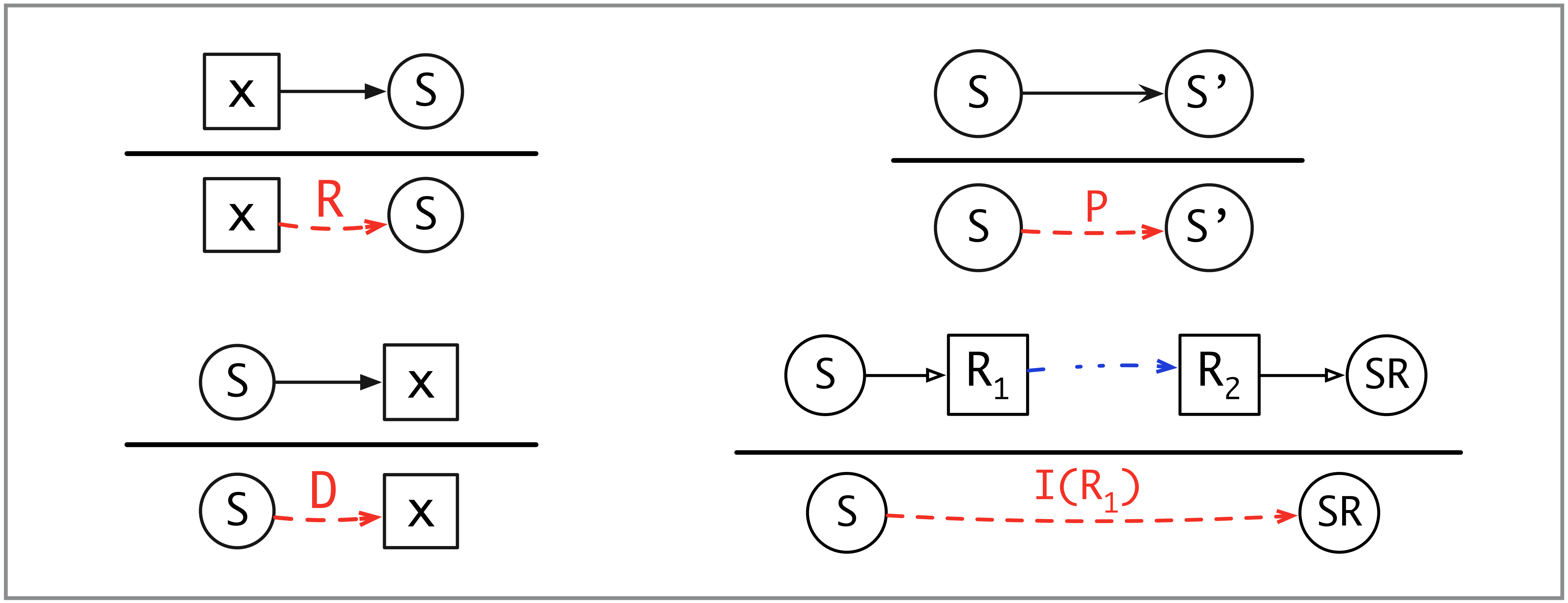
Transitive vs. Non-Transitive



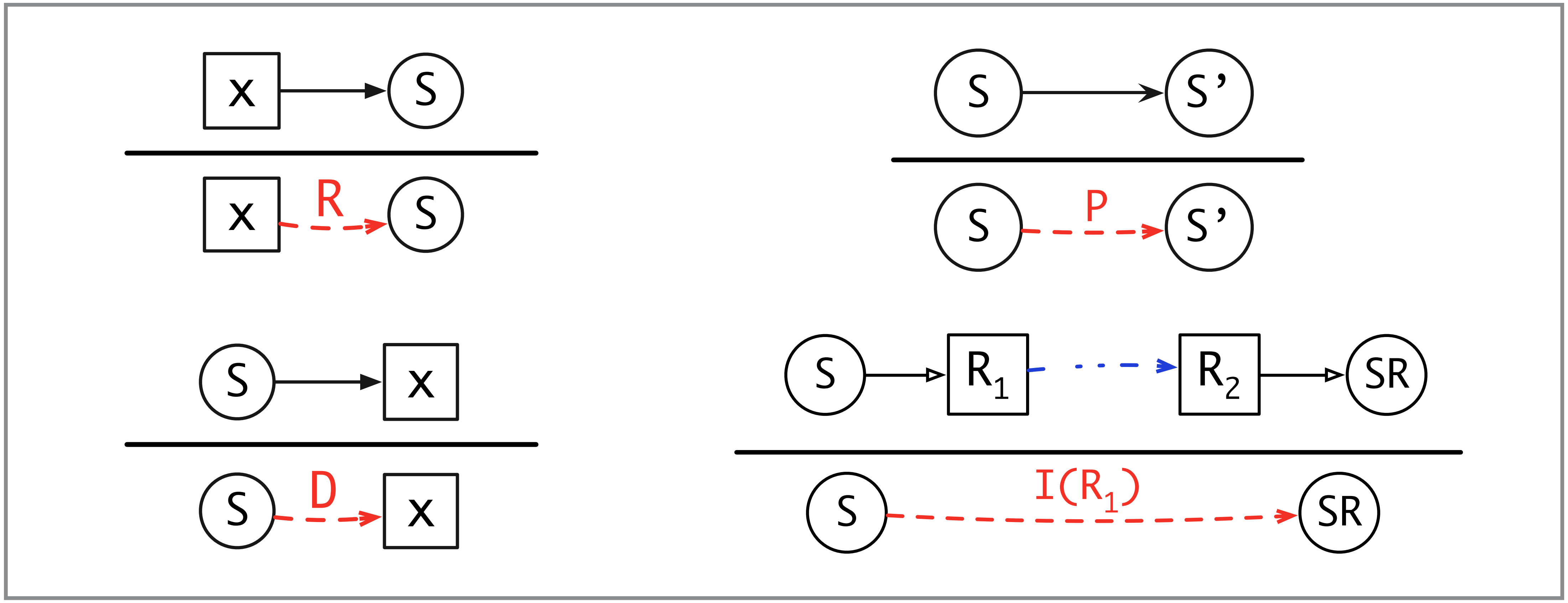
With transitive imports, a well formed path is `R.P*.I(_)*.D`

With non-transitive imports, a well formed path is `R.P*.I(_)?.D`

A Calculus for Name Resolution



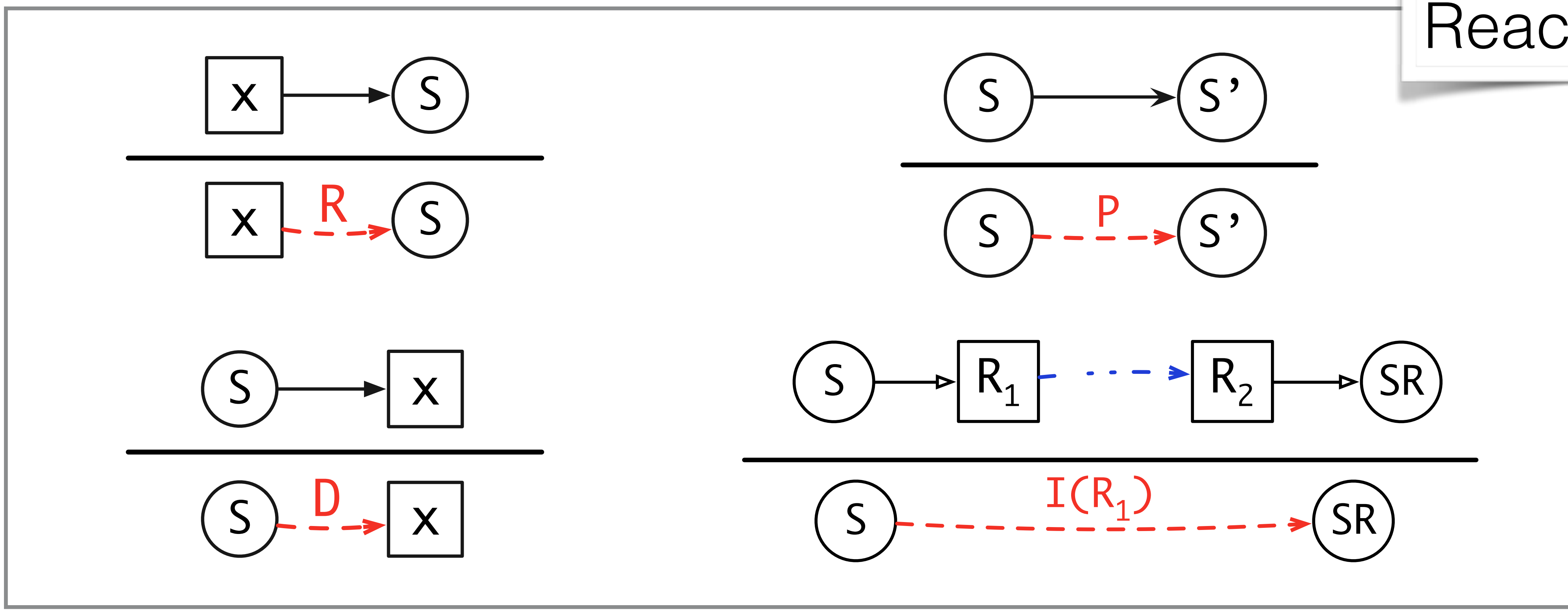
A Calculus for Name Resolution



Well formed path: R.P*.I(_)*.D

A Calculus for Name Resolution

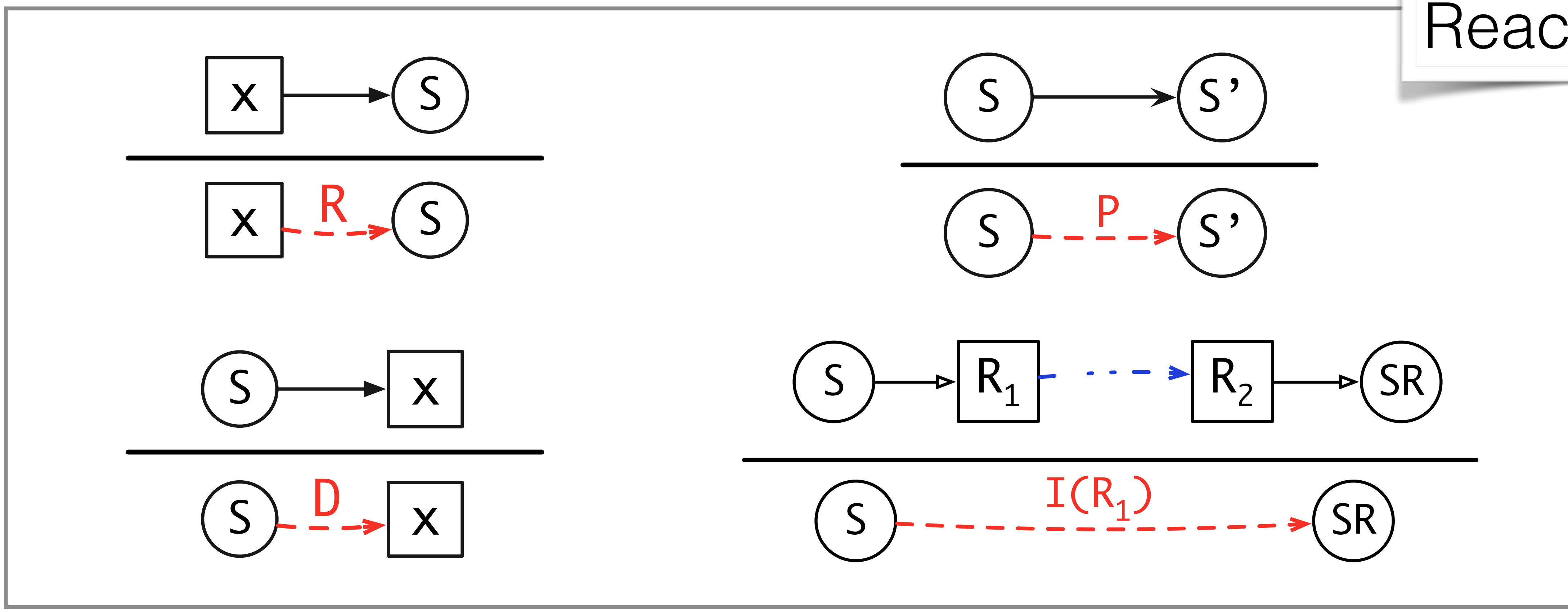
Reachability



Well formed path: R.P*.I(_)*.D

A Calculus for Name Resolution

Reachability



Well formed path: R.P*.I(_)*.D

$$\frac{}{D < P.p} \qquad \frac{}{I(\_).p' < P.p}$$

$$\frac{p < p'}{s.p < s.p'} \qquad \frac{}{D < I(\_).p'}$$

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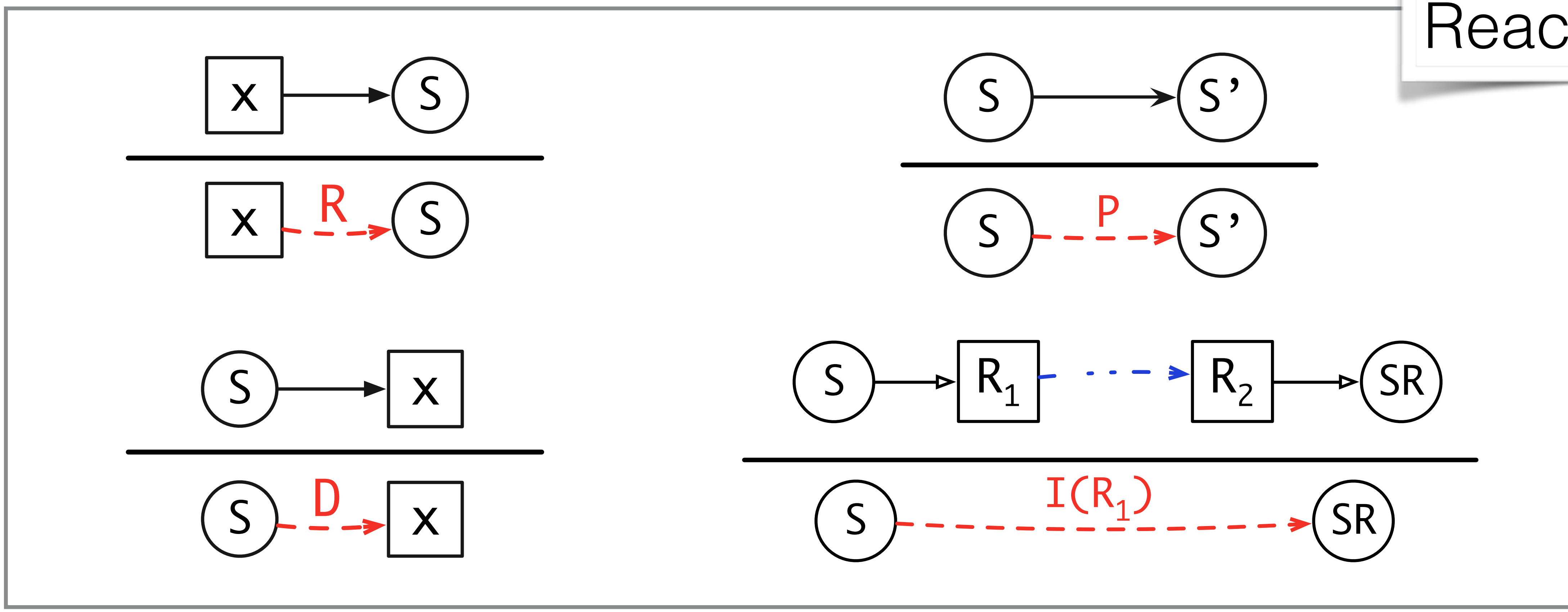

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Visibility

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$$\frac{p < p'}{s.p < s.p'} \qquad \frac{}{D < I(\_).p'}$$

Theory of Scope Graphs

Scope graphs, formally

$x_i^{\mathsf{D}}{:}S$ declaration with name x at position i with optional associated scope S

x_i^{R} reference with name x at position i

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$\mathcal{G}$: scope graph
$\mathcal{S}(\mathcal{G})$: scopes S in $\mathcal{G}$
$\mathcal{D}(S)$: declarations $x_i^{\mathsf{D}}{:}S'$ in S
$\mathcal{R}(S)$: references x_i^{R} in S
$\mathcal{I}(S)$: imports x_i^{R} in S
$\mathcal{P}(S)$: parent scope of S

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- $\mathcal{P}(S)$ is a partial function
- The parent relation is well-founded
- Each x_i^{R} and x_i^{D} appears in exactly one scope S

Resolution calculus, formally

Resolution paths

$$\frac{x_i^{\mathsf{R}} \in \mathcal{R}(S) \quad \{x_i^{\mathsf{R}}\} \cup \mathbb{I} \vdash p : S \longmapsto x_j^{\mathsf{D}}}{\mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}}}$$

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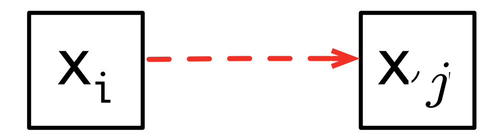

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$$\boxed{\mathsf{x}_i} \dashrightarrow \boxed{\mathsf{x}_{\prime j}}$$

Visibility paths

$$\frac{\mathbb{I} \vdash p : S \rightarrowtail x_i^{\mathsf{D}} \qquad \forall j, p'(\mathbb{I} \vdash p' : S \rightarrowtail x_j^{\mathsf{D}} \Rightarrow \neg(p' < p))}{\mathbb{I} \vdash p : S \longmapsto x_i^{\mathsf{D}}}$$

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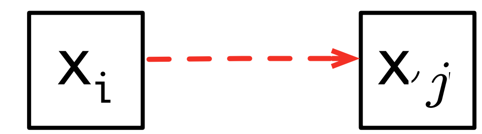

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Reachability paths

$$\frac{\vdots}{\mathbb{I} \vdash p : S \rightarrowtail x_i^{\mathsf{D}}}$$

Resolution calculus, formally

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$$\frac{x_i^{\mathsf{R}} \in \mathcal{R}(S) \qquad \{x_i^{\mathsf{R}}\} \cup \mathbb{I} \vdash p : S \longmapsto x_j^{\mathsf{D}}}{\mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}}}$$

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...may include import steps

$$\frac{y_i^{\mathsf{R}} \in \mathcal{I}(S_1) \setminus \mathbb{I} \qquad \mathbb{I} \vdash p : y_i^{\mathsf{R}} \longmapsto y_j^{\mathsf{D}}{:}S_2}{\mathbb{I} \vdash \mathbf{I}(y_i^{\mathsf{R}}, y_j^{\mathsf{D}}{:}S_2) : S_1 \longrightarrow S_2}$$

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$$\boxed{x_i} \dashrightarrow \boxed{x_j}$$

Visibility paths

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Well-founded!

...may include import steps

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  - and reachability paths can have cycles

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- But there is a terminating resolution algorithm
  - for path well-foundedness `RP*I*D`
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Resolution Algorithm

- Resolution calculus says how to validate paths but not how to find them
  - and reachability paths can have cycles
- But there is a terminating resolution algorithm
  - for path well-foundedness `RP*I*D`
  - and path ordering `D<I<P`
- Uses familiar notions of environments and shadowing

Resolution Algorithm v1

$$
\begin{aligned}
E_D(S) &:= \mathcal{D}(S) \\
E_P(S) &:= E_V(\mathcal{P}(S)) \\
E_I(S) &:= \bigcup \left\{ E_L(S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \land y_j^{\mathsf{D}}{:}S_y \in \mathit{Resolve}(y_i^{\mathsf{R}}) \right\} \\
E_L(S) &:= E_D(S) \triangleleft E_I(S) \\
E_V(S) &:= E_L(S) \triangleleft E_P(S) \\
\mathit{Resolve}(x_i^{\mathsf{R}}) &:= \{ x_j^{\mathsf{D}} \mid \exists S \ s.t.\ x_i^{\mathsf{R}} \in \mathcal{R}(S) \land x_j^{\mathsf{D}} \in E_V(S) \}
\end{aligned}
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where $E_1 \triangleleft E_2 := E_1 \cup \{ x_i^{\mathsf{D}} \in E_2 \mid \nexists\ x_{i'}^{\mathsf{D}} \in E_1 \}$

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- still need to incorporate “seen imports”

Resolution Algorithm v2

$$
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E_P[\mathbb{I}](S) \quad &:= E_V[\mathbb{I}](\mathcal{P}(S)) \\
E_I[\mathbb{I}](S) \quad &:= \bigcup \left\{ E_L[\mathbb{I}](S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \setminus \mathbb{I} \wedge y_j^{\mathsf{D}}{:}S_y \in \mathit{Resolve}[\mathbb{I}](y_i^{\mathsf{R}}) \right\} \\
E_L[\mathbb{I}](S) \quad &:= E_D[\mathbb{I}](S) \triangleleft E_I[\mathbb{I}](S) \\
E_V[\mathbb{I}](S) \quad &:= E_L[\mathbb{I}](S) \triangleleft E_P[\mathbb{I}](S) \\
\mathit{Resolve}[\mathbb{I}](x_i^{\mathsf{R}}) &:= \{ x_j^{\mathsf{D}} \mid \exists S \ s.t. \ x_i^{\mathsf{R}} \in \mathcal{R}(S) \wedge x_j^{\mathsf{D}} \in E_V[\{x_i^{\mathsf{R}}\} \cup \mathbb{I}](S) \}
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- but still might not terminate due to cycles (e.g. consider a scope that imports itself)

Resolution Algorithm v3

$$E_D[\mathbb{I},\mathbb{S}](S) := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ \mathcal{D}(S) \end{cases}$$

$$E_P[\mathbb{I},\mathbb{S}](S) := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ E_V[\mathbb{I},\{S\} \cup \mathbb{S}](\mathcal{P}(S)) \end{cases}$$

$$E_I[\mathbb{I},\mathbb{S}](S) := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ \bigcup \Big\{ E_L[\mathbb{I},\{S\} \cup \mathbb{S}](S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \setminus \mathbb{I} \wedge y_j^{\mathsf{D}}{:}S_y \in \mathit{Resolve}[\mathbb{I}](y_i^{\mathsf{R}}) \Big\} \end{cases}$$

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Lemma: visibility paths never have cycles

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Lemma: visibility paths never have cycles

Theorem: algorithm is sound and complete for calculus:

$$(x_j^{\mathsf{D}} \in \mathit{Res}[\mathbb{I}](x_i^{\mathsf{R}})) \iff (\exists p \ s.t. \ \mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}})$$

Language-independent α-equivalence

Program similarity

$\texttt{P} \simeq \texttt{P'}$ if have same AST ignoring identifier names

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Position equivalence

$$\frac{\boxed{\texttt{x}_\texttt{i}} \dashrightarrow \boxed{\texttt{x}_{\texttt{i'}}}}{i \overset{\texttt{P}}{\sim} i'} \qquad \frac{i' \overset{\texttt{P}}{\sim} i}{i \overset{\texttt{P}}{\sim} i'} \qquad \frac{i \overset{\texttt{P}}{\sim} i' \quad i' \overset{\texttt{P}}{\sim} i''}{i \overset{\texttt{P}}{\sim} i''} \qquad \frac{}{i \overset{\texttt{P}}{\sim} i}$$

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Alpha equivalence

$$\texttt{P1} \overset{\alpha}{\approx} \texttt{P2} \triangleq \texttt{P1} \simeq \texttt{P2} \land \forall\ e\ e',\ e \overset{\texttt{P1}}{\sim} e' \Leftrightarrow e \overset{\texttt{P2}}{\sim} e'$$

(with some further details about free variables)

Preserving ambiguity

module $\mathtt{A}_{1}$ {
  def $\mathtt{x}_{2}$:= 1
}
module $\mathtt{B}_{3}$ {
  def $\mathtt{x}_{4}$:= 2
}
module $\mathtt{C}_{5}$ {
  import $\mathtt{A}_{6}$ $\mathtt{B}_{7}$;
  def $\mathtt{y}_{8}$:= $\mathtt{x}_{9}$
}
module $\mathtt{D}_{10}$ {
  import $\mathtt{A}_{11}$;
  def $\mathtt{y}_{12}$:= $\mathtt{x}_{13}$
}
module $\mathtt{E}_{14}$ {
  import $\mathtt{B}_{15}$;
  def $\mathtt{y}_{16}$:= $\mathtt{x}_{17}$
}

P1

module $\mathtt{AA}_{1}$ {
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module $\mathtt{BB}_{3}$ {
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  import $\mathtt{AA}_{6}$ $\mathtt{BB}_{7}$;
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}
module $\mathtt{E}_{14}$ {
  import $\mathtt{BB}_{15}$;
  def $\mathtt{v}_{16}$:= $\mathtt{z}_{17}$
}

P2

module $\mathtt{A}_{1}$ {
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}
module $\mathtt{B}_{3}$ {
  def $\mathtt{x}_{4}$:= 2
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}

P3

P1 $\overset{\alpha}{\approx}$ P2   P2 $\overset{\alpha}{\not\approx}$ P3

Applying Scope Graphs

(ongoing work)

Validation

Validation

- We have modeled a large set of example binding patterns
  - definition before use
  - different let binding flavors
  - recursive modules
  - imports and includes
  - qualified names
  - class inheritance
  - partial classes
  - …

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- We have modeled a large set of example binding patterns
    - definition before use
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    - class inheritance
    - partial classes
    - …
- Next goal: fully model some real languages
    - Java
    - ML
    - …

Generating Scope Graphs from AST

$$
\begin{array}{rcll}
[\![ds]\!]^{prog} & := & P(S) := \bot \ \wedge\ [\![ds]\!]^{decl^*}_{S} & (\textit{new } S) \\
[\![\texttt{module}\ X_i\ \{\, ds \,\}]\!]^{decl}_{s} & := & X_i^{\mathsf{D}}{:}S' \in \mathcal{D}(s) \ \wedge\ P(S') := s \ \wedge\ [\![ds]\!]^{decl^*}_{S'} & (\textit{new } S') \\
[\![\texttt{import}\ Xs\,.\,X_i]\!]^{decl}_{s} & := & X_i^{\mathsf{R}} \in \mathcal{I}(s) \ \wedge\ [\![Xs\,.\,X_i]\!]^{qid}_{s} & \\
[\![\texttt{def}\ b]\!]^{decl}_{s} & := & [\![b]\!]^{bind}_{s,s} & \\
[\![x_i = e]\!]^{bind}_{s_r,s_d} & := & x_i^{\mathsf{D}} \in \mathcal{D}(s_d) \ \wedge\ [\![e]\!]^{exp}_{s_r} & \\
[\![x_i]\!]^{qid}_{s} & := & x_i^{\mathsf{R}} \in \mathcal{R}(s) & \\
[\![\texttt{fun}\ (x_i : t)\ \{\, e \,\}]\!]^{exp}_{s} & := & P(S') := s \ \wedge\ x_i^{\mathsf{D}} \in \mathcal{D}(S') \ \wedge\ [\![e]\!]^{exp}_{S'} & (\textit{new } S') \\
[\![\texttt{letrec}\ bs\ \texttt{in}\ e]\!]^{exp}_{s} & := & P(S') := s \ \wedge\ [\![bs]\!]^{bind^*}_{S',S'} \ \wedge\ [\![e]\!]^{exp}_{S'} & (\textit{new } S') \\
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[\![e_1\ e_2]\!]^{exp}_{s} & := & [\![e_1]\!]^{exp}_{s} \ \wedge\ [\![e_2]\!]^{exp}_{s} &
\end{array}
$$

generate smallest graph satisfying constraints

Binding gives Types

Static type-checking (or inference) is one obvious client for name resolution

In many cases, we can perform resolution **before** doing type analysis

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def x : int = 6
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def x : int = 6
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```

Types give Binding

But sometimes we need types **before** we can do name resolution

```
record A₁ { x₁ : int }
record B₁ { a₁ : A₂ ; x₂ : bool}


def z₁ : B₂ = ...

def y₁ = z₂.x₃

def y₂ = z₃.a₂.x₄
```

Types give Binding

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$$\texttt{record } A_1 \ \{\ x_1 : \texttt{int}\ \}$$

$$\texttt{record } B_1 \ \{\ a_1 : A_2\ ;\ x_2 : \texttt{bool}\}$$

$$\texttt{def } z_1 : B_2 = \ldots$$

$$\texttt{def } y_1 = z_2.x_3$$

$$\texttt{def } y_2 = z_3.a_2.x_4$$

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$$\texttt{record}\ A_1\ \{\ x_1 : \texttt{int}\ \}$$
$$\texttt{record}\ B_1\ \{\ a_1 : A_2\ ;\ x_2 : \texttt{bool}\}$$

$$\texttt{def}\ z_1 : B_2 = \ ...$$

$$\texttt{def}\ y_1 = z_2.x_3$$

$$\texttt{def}\ y_2 = z_3.a_2.x_4$$

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Our approach: interleave **partial** name resolution with type resolution (also using constraints)

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- Dynamic analogs to static scope graphs

Use Scope Graphs to Describe Name Resolution

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